B.SC. PHYSICS SEMESTER – V CORE - V ELECTRICITY AND MAGNETISM

UNIT I

Principle of a capacitor – energy stored in a capacitor – energy density – change in energy due to dielectric slab – force of attraction between plates of a charged capacitor – capacitance of a spherical and cylindrical capacitors – types of capacitors – quadrant electrometer – measurement of potential, ionization current and dielectric constant.

UNIT II

Carey–Foster Bridge – theory – temperature coefficient of resistance – potentiometer – calibration of ammeter and high range voltmeter – thermoelectricity – laws of thermo e.m.f., intermediate metals, intermediate temperature – measurement of thermo e.m.f. using potentiometer–Peltier effect and Peltier coefficient – Thomson effect and Thomson coefficient – relation between π and σ – thermo electric diagrams and its uses.

UNIT III

Magnetic induction due to a straight conductor carrying current – magnetic induction on the axis of a solenoid – moving coil ballistic galvanometer – damping correction – determination of absolute capacity of a condenser – self – inductance by Anderson's Bridge method – experimental determination of mutual inductance – coefficient of coupling.

UNIT IV

Transient current – growth and decay of current in a circuit containing resistance and inductance – growth and decay of charge in a circuit containing resistance and capacitance – measurement of high resistance by leakage – growth and decay of charge in a LCR circuit – condition for the discharge to be oscillatory – frequency of oscillation.

UNIT V

Alternating current – peak, average and RMS value of current and voltage – form factor – ac circuit containing resistance and inductance – choke coil – ac circuit containing resistance and capacitance – series and parallel resonance circuits –Q factor – power in an ac circuit containing LCR – Wattless current – Transformer – construction, theory and uses – energy loss – skin effect.

BOOKS FOR STUDY:

1. Brijlal and Subramaniam, Electricity and Magnetism, S. Chand & Co, New Delhi (2016)
2. R. Murugeshan, Electricity and Magnetism, S. Chand & Co, New Delhi (2016)
3. Hugh D. Young and Roger A. Freedman, Sears & Zemansky's University Physics with Modern Physics,14th Edition (2015)

BOOKS FOR REFERENCE:

1. D. N. Vasudeva, Electricity and Magnetism, S. Chand & Co, New Delhi (2016)
2. K. K. Tewari, Electricity and Magnetism, S. Chand & Co, New Delhi (2016)
3. Hugh D. Young and Roger A. Freedman, Sears & Zemansky's University Physics with Modern Physics,14th Edition (2015)

ELECTRICITY AND MAGNETISM

UNIT- I

Principle of a capacitor - energy stored in a capacitor - energy density - change in energy due to dielectric slab - force of attraction between plates of a charged capacitor - capacitance of a spherical and cylindrical capacitors - types of capacitors - electrometers - Kelvin's attracted disc electrometer - quadrant electrometer - measurement of potential, ionization current and dielectric constant (solid).

Topics – UNIT 1:

I CAPACITORS

1. **Capacitance – Definition, Types of capacitance**
2. **Principle of a capacitor**
3. **Energy stored in a capacitor**
4. **Energy density of a charged capacitor**
5. **Change in energy due to Dielectric Slab**
6. **Loss of energy on sharing of charges between two capacitors**
7. **Force of attraction between plates of a charged parallel plate capacitor**
8. **Capacitance of a Spherical capacitor (outer sphere earthed)**
9. **Capacitance of a Spherical capacitor (inner sphere earthed)**
10. **Capacitance of a Cylindrical capacitor**
11. **Types of capacitors**
    a. **Guard ring capacitor**
    b. **Mica capacitor**
    c. **Electrolytic capacitor**
    d. **Variable capacitor**
12. **Uses of capacitors**

II Electrometers

13. **Kelvin's attracted disc electrometer**
14. **Quadrant electrometer**
    a. **measurement of potential,**
    b. **ionization current**
    c. **dielectric constant (solid).**

1. Capacity of a capacitor

The capacity of a conductor is also defined as the ratio of the charge given (Q) to the increase in the potential of the conductor.

The Capacitance of a conductor

C= Q/V.

Unit - Farad

A conductor has a capacitance of one farad, if a charge of 1 coulomb given to it raises its potential by 1 volt

$$1\mu F = 10^{-6} F$$

$$1pF = 10^{-12} F$$

- If a charge q is given to an isolated conductor, its voltage is increased by an amount V.
- The ratio Q/V is called the **Capacitance of a conductor**. It is denoted by the symbol **C**

For a given conductor the ratio Q/V is

- independent of Q
- depends only on the size and shape of the conductor.

The common word used in our daily life for capacitors is Condensers which we use in most of our electric appliances.

Types of capacitors

Parallel plate capacitors are those in which conductors used are simple parallel plates.

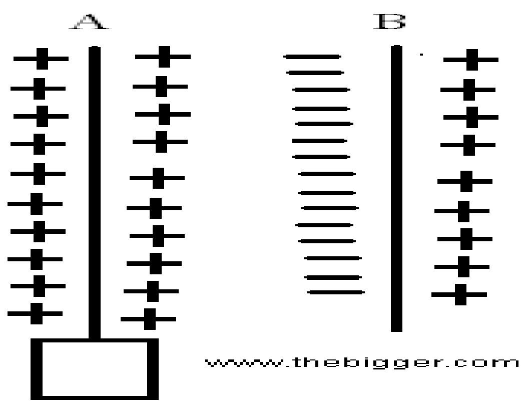
Spherical conductors are those in which spherical conductors are used.

Third type of conductors are those in which conductors used are of **cylindrical type**.

2. Principle of a capacitor

A small device used to store huge amount of electric charge in a small room is called capacitor.

Take an insulated metal plate A. Charge the plate to its maximum potential. Now take another insulated plate B. Take the plate B nearer to plate A. You will observe that negative charge will be produce on the plate near to plate A and the same amount of positive charge will be produced on the other side of plate B.



Now the plate B will start affecting the plate A slowly. The negative charge will start decreasing the electric potential of plate A. But positive charge helps in increasing the potential. But the effect of negative charge is much more than that of the positive because the negative side of plate is near to the plate A. So potential of A will start decreasing and it can be charged again to raise its potential to maximum.

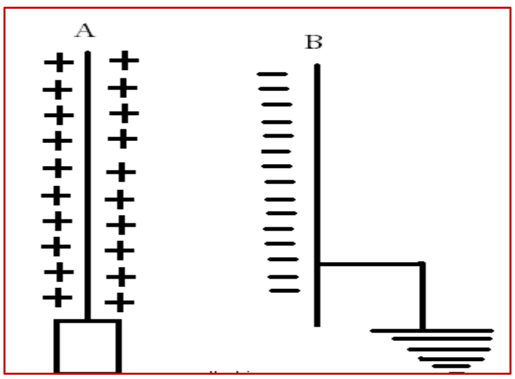
From the above discussion the result is this that the charge carrying capacity of a conductor can be increased by bringing an uncharged conductor in its nearby area. It is shown in the figure above.

Let us take an another discussion over it.

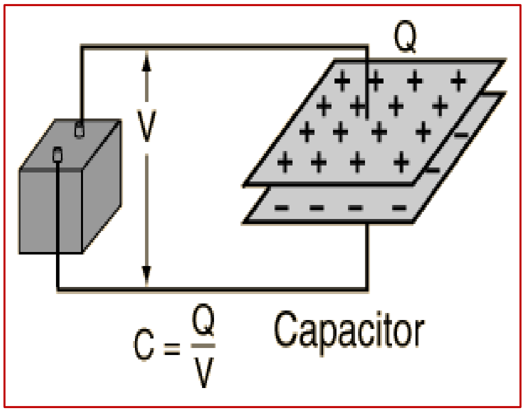
Now in this case connect the plate B to earth. All the positive charge present on plate B will go into the earth. So, only negative charge will remain on the plate B. So the electric potential of plate A will become less to a greater extent. . So as a result A will want much more charge to gain its lost potential due to the effect of the negative charge present on plate B.

Note: By taking an uncharged conductor near an insulated conductor,

capacitance of the insulated conductor can be increased to a larger amount.



3. Energy stored in a charged capacitor



Let q is the amount of charge stored when the whole battery voltage (V) appears across the capacitor. If an additional charge dq is given to the plates, the workdone by the battery is given by dW = V dq

Since $V = \frac{q}{C}$

$$dW = \frac{q}{C}\, dq$$

Total Work done to charge a capacitor to a charge Q is

$$W = \int dW = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2}\frac{Q^2}{C}$$

We know that Q = CV

$$W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

- This work done $\mathbf{U = \frac{1}{2} CV^2}$ is stored as electrostatic potential energy in the capacitor.
- This energy can be recovered if the capacitor is allowed to discharge.

4. Energy density of a charged capacitor

Consider a parallel plate capacitor of area A and plate separation d.

Energy of the capacitor $U = \frac{1}{2}CV^2$

For a parallel plate capacitor, the capacitance C = (ε_0A) / d

Volume of the space between the plates = Ad

(Volume = length *breadth *width = Area * Thickness)

Energy density (u) is the potential energy per unit volume

$$u = \frac{U}{V} = \frac{1}{2}\frac{CV^2}{V} = \frac{1}{2}\frac{\varepsilon_0 A V^2}{d\ (Ad)} = \frac{1}{2}\frac{\varepsilon_0 V^2}{d^2}$$

W.K.T (E = V/d)

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Thus we can associate an electrostatic energy density $u = \frac{1}{2}\varepsilon_0 E^2$ **with every point in space where an electric field E exists**

5. Change in Energy due to Dielectric Slab

Change in energy of a Parallel Plate Capacitor on introduction of a dielectric slab of relative permittivity ε_r between the plates

Case 1. When the charge remains the same

When the capacitor is charged by connecting the plates to the + ve and – ve ends of a battery, and the battery is then withdrawn,

(a) Without the slab, energy stored

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\varepsilon_0 A}$$

(b) Let the space between the parallel plates be introduced by a slab of thickness d and relative permittivity ε_r .

Now capacitance $C' = \frac{\varepsilon_0 \varepsilon_r}{d} A$

With the slab, the energy stored is

$$U' = \frac{q^2}{2c'} = \frac{q^2 d}{2\varepsilon_0 \varepsilon_r A}$$

Thus the energy will increase by a factor 1/ ε_r

When a slab of thickness t, (t < d) is introduced,

$$c' = \frac{\epsilon_0 A}{(d-t) + t/\varepsilon_r}$$

$$U' = \frac{q^2}{2c'} = \frac{q^2 \left[(d-t) + t/\varepsilon_r\right]}{2\epsilon_0 A} = \frac{q^2}{2\epsilon_0 A}\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right)\right]$$

Reduction in energy stored = **U – U’**

$$= \frac{q^2\, t\,(1 - \frac{1}{\varepsilon_r})}{2\epsilon_0 A}$$

Case: (ii) When the potential remains the same.

e.g when the battery is kept connected to the plates.

(a) without the slab, energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d} V^2 = \frac{\epsilon_0 AV^2}{2d}$$

(b) On introducing a dielectric slab of thickness d,

$$U' = \frac{1}{2} C'V^2 = \frac{1}{2}\frac{\epsilon_0\epsilon_r A}{d} V^2 = \frac{\epsilon_0\epsilon_r AV^2}{2d}$$

Thus the energy will increase by a factor ϵ_r.

When a slab of thickness t, (t < d), is introduced, energy stored

$$U' = \frac{1}{2} C'V^2 = \frac{\epsilon_0 AV^2}{2\left[d-t\left(1-\frac{1}{\varepsilon_r}\right)\right]}$$

In this case, U’ > U.

6. Loss of energy on sharing of charges between two capacitors:

Consider two capacitors of capacitances C_1 and C_2 charged to potentials V_1 and V_2. When they are joined by a wire, they attain a common potential V.

$$V = \frac{\text{Total charge}}{\text{Total Capacitance}} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Total energy of the two capacitors before contact

$$U_1 = \frac{1}{2}\ C_1V_1^2 + \frac{1}{2}\ C_2V_2^2$$

Total energy of the two capacitors after contact

$$U_2 = \frac{1}{2}\ (C_1 + C_2)\ V^2 = \frac{1}{2}\ (C_1 + C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right]^2$$

$$U_2 = \frac{1}{2}\ \frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

Loss of energy due to contact,

$$U_1 - U_2 = \frac{1}{2}\ C_1V_1^2 + \frac{1}{2}\ C_2V_2^2 - \frac{1}{2}\ \frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$= \frac{1}{2\,(C_1+C_2)}\ [(C_1 + C_2)(\ C_1V_1^2 +\ C_2V_2^2) - (C_1V_1 + C_2V_2)^2]$$

$$= \frac{1}{2\,(C_1+C_2)}\ [(\ C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 +\ \ C_2^2V_2^2) - (C_1^2V_1^2 +\ C_2^2V_2^2 +\ 2C_1V_1C_2V_2)]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1+C_2}[V_1^2 + V_2^2 - 2\ V_1V_2]$$

$$= \frac{C_1C_2\ (V_1-V_2)^2}{2\ (C_1+C_2)}$$

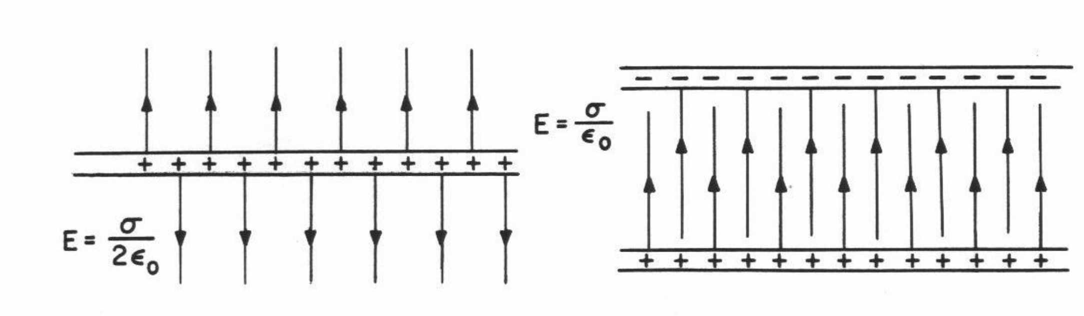
Since $(V_1 - V_2)^2$ is always positive, U_2 must be less than U_1. Hence there is a loss of energy on sharing the charges. The loss of energy appears partly as heat in the connecting wire and partly as light and sound if sparking occurs.

7. Force of attraction between plates of a charged parallel plate capacitor

Let q be the charge on each of the plates. A be the area and d be the distance between them. Let σ be the surface charge density and E the intensity of electric field between the plates.

$$\boldsymbol{E = \frac{\sigma}{\varepsilon_0}}$$

Casse (i). When the charge on the plates is constant.



Force of attraction per unit area between the two plates is equal to the outward electrical force per unit area on the surface of plate P. It is given by

$$p = \frac{\sigma^2}{2\varepsilon_0} = \frac{(q/A)^2}{2\varepsilon_0} = \frac{q^2}{2\varepsilon_0 A^2} = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{where } \left(\boldsymbol{E = \frac{\sigma}{\varepsilon_0}}\right)$$

Force of attraction between the plates = $F = Ap = \frac{q^2}{2\varepsilon_0 A^2} A = \frac{1}{2}\varepsilon_0 E^2 A$

Case (ii): When the P.D between the plates remain constant:

In this case, a battery of EMF V volts is connected across the plates of a capacitor.

Now, E = V/d

$$F = \frac{1}{2}\varepsilon_0 E^2 A = \frac{1}{2}\varepsilon_0 \left(V/d\right)^2 A$$

Thus we can find F if we know $\sigma, or\ E\ or\ V$.

capacitance depends on the geometry of the conductors and the permittivity of the medium separating them. A capacitor is a device for storing charge.

4.2. Capacitance of a Spherical Capacitor (outer sphere earthed)

Let A and B be two concentric metal spheres of radii a and b respectively with air as the intervening medium (Fig. 4.2). The outer sphere B is earthed. A charge $+q$ is given to the inner sphere. The induced charge on the inner surface of the outer sphere is $-q$. P is a point at a distance r from the common centre O.

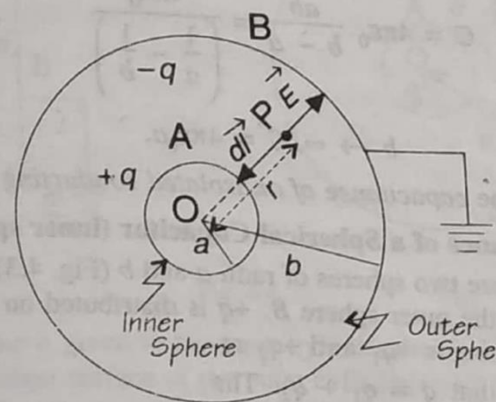


Fig. 4.2

Electric field at $P = \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)\hat{\mathbf{r}}$ ... (1)

where $\hat{\mathbf{r}}$ is the unit vector along $\overrightarrow{OP}$.

The potential difference between the spheres A and B is given by

$$V = -\int_b^a \mathbf{E}\cdot d\mathbf{l} \quad \text{... (2)}$$

Here, $d\mathbf{l}$ is the differential vector displacement along a path *from B* to *A*.

But $\mathbf{E}\cdot d\mathbf{l} = E\,dl\cos 180^\circ = -E\,dl.$

Further, in moving a distance dl in the direction of motion, we are moving in the direction of *r decreasing*, so that $dl = -dr$. Hence,

$$\mathbf{E}\cdot d\mathbf{l} = E\,dr.$$

Eq. (2) becomes $V = -\int_b^a E\,dr.$

Putting the value of E from Eq. (1), we get

$$V = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0}\left\{-\frac{1}{r}\right\}_b^a$$

$$= \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{a} - \frac{1}{b}\right\} = \frac{q}{4\pi\varepsilon_0}\,\frac{b-a}{ab}$$

$\therefore$ Capacitance of the spherical capacitor

$$C = \frac{q}{V} = \frac{q}{\left(\frac{q}{4\pi\varepsilon_0}\right)\left(\frac{b-a}{ab}\right)} = 4\pi\varepsilon_0\,\frac{ab}{(b-a)} \quad \cdots (3)$$

Note. Eq. (3) can be written in the form

$$C = 4\pi\varepsilon_0\,\frac{ab}{b-a} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

When $b \to \infty$, $C = 4\pi\varepsilon_0 a$.

This is the *capacitance of an isolated conducting sphere* of radius a.

4.3. Capacitance of a Spherical Capacitor (inner sphere earthed)

A and B are two spheres of radii a and b (Fig. 4.3). Suppose a charge $+q$ is given to the outer sphere B. $+q$ is distributed on its inner and outer surfaces by amounts $+q_1$ and $+q_2$ respectively, so that $q = q_1 + q_2$. The charge $+q_1$ on the inner surface of B induces a charge $-q_1$ (bound charge) on the outer surface of A and charge $+q_1$ on the inner surface of A. The charge $+q_1$ on the inner surface of A, being free, leaks to the earth.

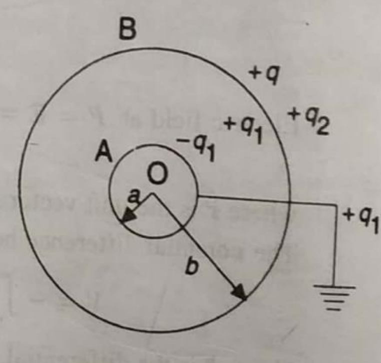


Fig. 4.3

The two spheres now behave as two capacitors connected in parallel.

(*i*) The inner sphere of radius a and the inner surface of outer sphere form a capacitor of capacitance

$$C_1 = \frac{4\pi\varepsilon_0 ab}{b-a} \quad \text{(if the dielectric is air)}$$

(*ii*) The outer surface of B and the earth form a capacitor of capacitance

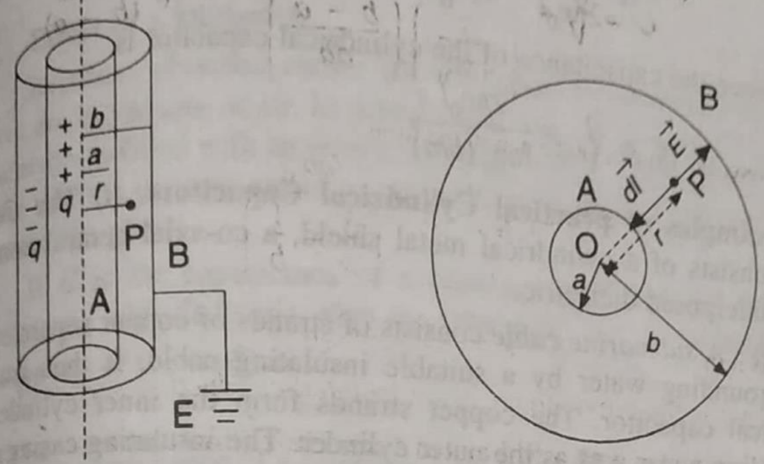
$$C_2 = 4\pi\varepsilon_0 b.$$

Total capacitance $C = C_1 + C_2 = \frac{4\pi\varepsilon_0 ab}{(b-a)} + 4\pi\varepsilon_0 b$

$$\therefore \quad C = \frac{4\pi\varepsilon_0 b^2}{b-a}$$

4.4. Capacitance of a Cylindrical Capacitor

Consider a cylindrical capacitor formed by two coaxial cylinders A and B of radii a and b respectively and each of length l. Air is the medium between A and B. The outer cylinder B is earthed (Fig. 4.4 a).



(a) Fig. 4.4 (b)

If a charge $+q$ is given to the inner cylinder,then an equal charge $-q$ is induced on the inner surface of the outer cylinder and a charge $+q$ on the outer surface of the outer cylinder. The charge $+q$ induced on the outer surface of the outer cylinder flows to the earth.

The electric field at a point P in the space between the two cylinders at a distance r from the axis is

$$E = \frac{1}{2\pi\varepsilon_0 l} \frac{q}{r}. \quad \text{... (1)}$$

The potential difference V between the cylinders A and B is

$$V = -\int_b^a \mathbf{E} \cdot d\mathbf{l} \quad \text{... (2)}$$

Here, $d\mathbf{l}$ is the vector displacement along a path from B to A (Fig. 4.4 b).

Now, $\mathbf{E}$ is radially outward and $d\mathbf{l}$ is inward. Therefore

$$\mathbf{E} \cdot d\mathbf{l} = E\,dl \cos 180^\circ = -E\,dl.$$

As we move a distance dl from B to A, we move in the direction of decreasing r. So $dl = -dr$. Thus

$$\mathbf{E} \cdot d\mathbf{l} = E\,dr.$$

Eq. (2) becomes, $V = -\int_b^a E\,dr$

$$= -\frac{q}{2\pi\varepsilon_0 l}\int_b^a \frac{dr}{r} \quad \text{[from Eq. (1)]}$$

$$= -\frac{q}{2\pi\varepsilon_0 l}\left\{\log_e r\right\}_b^a = -\frac{q}{2\pi\varepsilon_0 l}\left\{\log_e a - \log_e b\right\}$$

$$= \frac{q}{2\pi\varepsilon_0 l}\log_e \frac{b}{a}.$$

Hence the capacitance of the cylindrical capacitor is

$$C = \frac{q}{V} = \frac{2\pi\varepsilon_0 l}{\log_e (b/a)}$$

Examples of practical Cylindrical Capacitors. (*i*) The *Co-axial cable* consists of a cylindrical metal shield, a co-axial central conductor and an interposed dielectric.

(*ii*) A *submarine cable* consists of strands of copper separated from the surrounding water by a suitable insulating cable. It thus acts as a cylindrical capacitor. The copper strands form the inner cylinder. The surrounding water acts as the outer cylinder. The insulating casing acts as the dielectric.

4.13. Types of Capacitors.

(*a*) **Guard Ring Capacitor.** In a parallel plate capacitor, the electric field between the plates is not uniform near the edges. This is called the "*edge effect*" or "*fringing*". The expression $C = \varepsilon_0 A/d$ is only approximate. This is avoided by using a guard ring. The circular insulated plate P is surrounded by a circular coplanar ring G. The inner diameter of G is slightly larger than the diameter of P (Fig. 4.11).

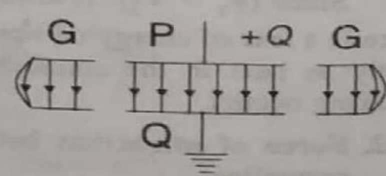


Fig. 4.11

The air gap between P and G is very small. The diameter of the plate Q is equal to the outer diameter of G. The field between P and Q is uniform throughout the common area between them. The irregularity in the field occurs at the outer edge of the guard ring. The effective area of the plate = A' = Area of the plate $P + \frac{1}{2}$ Area of the circular air gap between P and G. $C = \varepsilon_0 A'/d$. This is used as an absolute standard of capacitance.

(*b*) **Mica Capacitors.** A schematic diagram of a multiplate capacitor is shown in Fig. 4.12. It consists of a number of parallel plate capacitors in parallel with the alternate metallic foils fixed to one end each. Mica is used as the dielectric. The capacitance of such a system is $C = n\,\varepsilon_r \varepsilon_0 A/d$ where n is the number of capacitors grouped in parallel, A is the surface area of the plate and d is the thickness of each mica sheet.

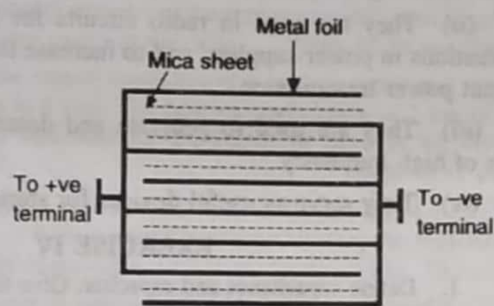


Fig. 4.12

(*c*) **Electrolytic Capacitor.** It consists of two aluminium electrodes A and C dipped in a solution of ammonium borate (Fig. 4.13). On passing a direct current, a very thin film of aluminium oxide is formed on the anode. This film is an insulator. The arrangement can now be used as a capacitor with the anode as one plate, the *solution* as the other plate, and the aluminium oxide film as dielectric. Since the dielectric layer is very thin, the capacitance of this arrangement is very large. This capacitor must be placed only in a D.C. circuit. It cannot be used in an A.C. circuit.

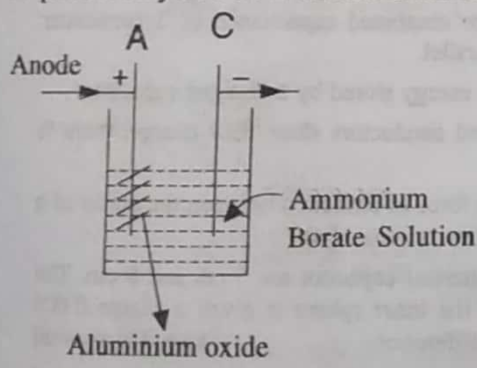


Fig. 4.13

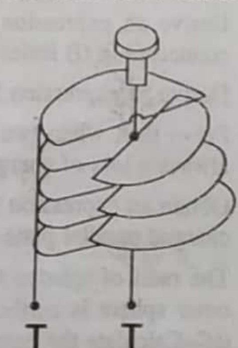


Fig. 4. 14

(*d*) **Variable Air capacitor.** It consists of two sets of metal plates, one fixed and the other movable (Fig. 4.14). The fixed set is semi-circular in shape. The movable set is like a cam and rotated with knobs. All the fixed plates are connected to one terminal. All the movable plates are connected to another terminal. Air is the dielectric. By rotating the knobs, the area of overlap between the two sets of plates is changed. Thus the capacitance of the capacitor changes. These capacitors are widely used in the tuning circuits of radio receivers.

Uses of Capacitors. (*i*) They are used in the ignition system of automobile engines for eliminating sparking.

(*ii*) They are used in radio circuits for tuning, to reduce voltage fluctuations in power supplies, and to increase the efficiency of alternating current power transmission.

(*iii*) They are used to generate and detect electromagnetic oscillations of high frequency.

(*iv*) They serve as useful devices for storing electric energy.

EXERCISE IV

1. Define capacitance and capacitor. Give the principle of a capacitor in detail.
2. A capacitor consists of two concentric spheres. Calculate the capacitance when

   (*a*) the inner sphere is charged and the outer sphere earthed,

   (*b*) the outer sphere is charged and the inner sphere earthed.
3. (*a*) Derive an expression for the capacitance per unit length of a capacitor consisting of two co-axial cylinders.

   (*b*) Derive an expression for the capacitance of a submarine cable of given length.
4. Derive an expression for the capacitance of a parallel plate capacitor. What will be the capacitance if the space between the plates is partially filled with a slab of thickness *d* and relative permittivity ε_r.
5. Derive an expression for the combined capacitance of 3 capacitors connected in (*i*) series (*ii*) parallel.
6. Derive an expression for the energy stored by a charged capacitor.
7. Prove that, when two charged conductors share their charge, there is always a loss of energy.
8. Obtain an expression for the force of attraction between the plates of a charged parallel plate capacitor.
9. The radii of spheres in a spherical capacitor are 5 cm and 8 cm. The outer sphere is earthed and the inner sphere is given a charge 0.005 μC. Calculate the potential difference. **[Ans.** 337.6 volts]
10. Calculate the capacitance of earth, viewed as a spherical conductor of radius 6370 km.

    [Ans. $C = 4\pi\varepsilon_0 R = 4 \times 3.14 \times (8.85 \times 10^{-12}) \times (6.370 \times 10^6)$
    $= 708 \times 10^{-6} F$]

5

Electrometers

5.1. Kelvin's The Attracted Disc or Absolute Electrometer

Principle. The instrument is based on the force of attraction between the plates of a charged parallel plate capacitor.

Construction. It consists of a guard ring, parallel plate air capacitor. P and Q are two circular metal plates parallel to each other (Fig. 5.1). A guard ring G surrounds the plate P. The plate P serves as the attracted disc. S is a spring attached to P. P can be moved up or down by using the micrometer screw N. P and G are joined by a metal wire and so they are at the same potential. Q can be raised or lowered by means of a micrometer screw M. The distance by which Q is raised or lowered is measured on the scale R.

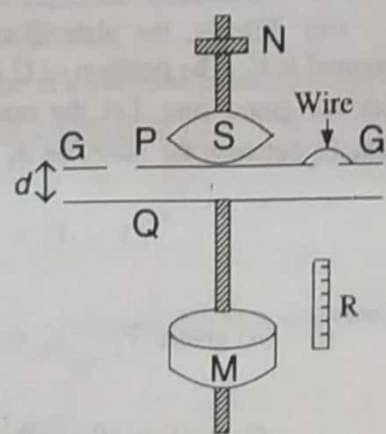


Fig. 5.1

Theory. Let the plates P and Q be connected to potentials V_p and V_q respectively. Let d be the distance between P and Q. Then, the electric field **E** between the plates P and Q is

$$E = (V_p - V_q)/d \quad \text{... (1)}$$

Therefore, the plate P will experience a force of attraction,

$$F = \frac{\varepsilon_0 E^2}{2} A = \frac{\varepsilon_0 A}{2} \frac{(V_p - V_q)^2}{d^2} \quad \text{... (2)}$$

where A is the "effective area" of the plate P.

$$\therefore \quad V_p - V_q = d\sqrt{\frac{2F}{\varepsilon_0 A}} \quad \text{... (3)}$$

Measurement of Potential Difference between two given points.

(*i*) The plates P and Q are connected to the earth so that the p.d.

between the plates is zero. A small mass m is placed on the plate P. Then P is depressed below the plane of G. The plate P is brought back to the same level as G by adjusting the screw N.

(*ii*) Then the mass is removed. The plate P goes above the level of G.

(*iii*) The plate P is connected to a potential V. Q is connected to one of the potential points, say V_1. The position of Q is adjusted with the help of the screw M, so that P comes in level with the guard ring. Then,

force of attraction between P and $Q = mg$.

The reading R_1 of the micrometer is noted. Let d_1 be the distance between P and Q. Then

$$V - V_1 = d_1 \sqrt{\frac{2mg}{\varepsilon_0 A}} \qquad \text{... (i)}$$

(*iv*) Finally, the plate Q is connected to the second point whose potential is V_2. The position of Q is again adjusted so that P comes in level with the guard ring. Let the reading on the micrometer be R_2. Let the distance between the plates be d_2. Then

$$V - V_2 = d_2 \sqrt{\frac{2mg}{\varepsilon_0 A}} \qquad \text{... (ii)}$$

(*i*) – (*ii*) gives, $V_2 - V_1 = (d_1 - d_2) \sqrt{\frac{2mg}{\varepsilon_0 A}}$

$$\therefore \quad V_2 - V_1 = (R_1 - R_2) \sqrt{\frac{2mg}{\varepsilon_0 A}} \qquad (\because d_1 - d_2 = R_1 - R_2)$$

Thus knowing mg, A and the change in micrometer screw reading, the p.d. between the two points is calculated.

P.D. is obtained in terms of absolute quantities like force, length and area. Hence it is called an *absolute electrometer*. It is comparatively less sensitive than the other forms of electrometers.

Determination of Relative Permittivity of a Material (in the form of a parallel slab).

A potential V_p is applied to P and a potential V_q to Q. The distance of Q from P is adjusted so that P is in a level with the guard ring. Let d be the distance between P and Q. Then the capacitance of the capacitor is

$$C = \varepsilon_0 A/d \qquad \text{... (i)}$$

The given slab of thickness t and of the same area as the plates, is placed on the plate Q. The effective air distance between the plates decreases by an amount $t\left(1 - \frac{1}{\varepsilon_r}\right)$. The force on the plate P increases and it moves down. The plate Q is moved down until the plate P goes back to its original position. Let x be the distance by which the plate Q is moved.

Now,

$$C = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right) + x\right]} \quad \text{... (ii)}$$

The capacitance of the capacitor in the two parts (with and without the slab) are equal. Therefore,

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{\varepsilon_r}\right) + x\right]} \quad \text{or } x = t - \frac{t}{\varepsilon_r}$$

$$\therefore \quad \varepsilon_r = \frac{t}{t - x} \quad \text{... (iii)}$$

The thickness of the slab t is measured by bringing the plates in contact with and without the slab in between these two. Thus ε_r is calculated.

5.2. The Quadrant electrometer

Construction. It consists of four similar hollow metallic quadrants AA and BB supported separately on amber insulating stands (Fig. 5.2). The opposite pairs of quadrants AA and BB are connected together by fine copper wires. A paddle-shaped aluminium needle with two wings CC is suspended symmetrically between the four quadrants by means of a torsion fibre made of phosphor bronze. The deflection of the needle is measured with the help of a mirror M using the lamp and scale arrangement. The whole arrangement is enclosed in an earthed brass case provided with glass windows to make observations. The base of the instrument is provided with levelling screws.

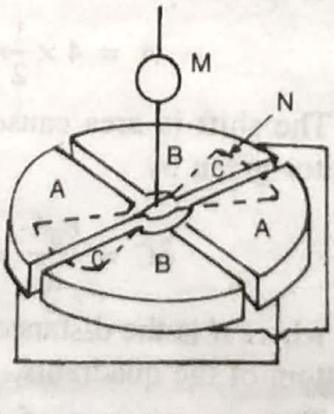


Fig. 5.2

Working. The needle N is usually kept at a constant high potential V_n by connecting it to the positive pole of a H.T. battery whose negative is earthed. When the two pairs of quadrants, AA and BB, are charged to the same potential, the needle rests symmetrically between them. If they are given different potentials, say V_a and V_b such that $V_a > V_b$, then the needle deflects from A quadrants to B quadrants. This deflection is opposed by the torsion of the suspension. At equilibrium, the torsional couple is equal and opposite to the deflecting couple. The deflection θ is proportional to the potential difference $(V_a - V_b)$.

Theory. Let V_n be the constant high potential applied to the needle (Fig. 5.3.). Let the pair of quadrants AA and BB be connected to points at potentials V_a and V_b respectively. Let $V_a > V_b$. The needle will get deflected through an angle θ from the pair AA (at higher potential) to the pair BB (at lower potential).

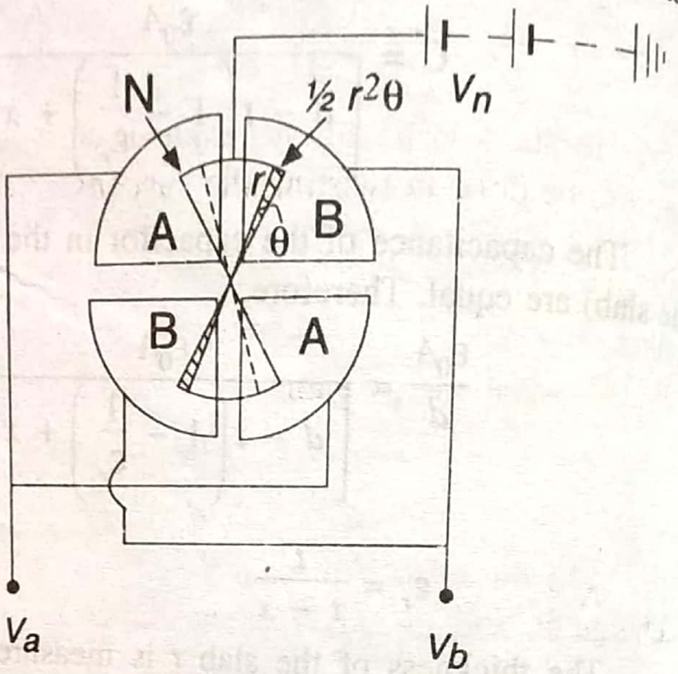


Fig. 5.3

The quadrants AA and the needle N form a parallel-plate capacitor at a potential difference $(V_n - V_a)$. The quadrants BB and the needle form another capacitor at a potential difference $(V_n - V_b)$.

Let r be the radius of the needle. With the deflection of the needle through an angle θ, a surface area $\frac{1}{2}r^2\theta$ of the needle is transferred from the quadrants AA to BB. Since the needle has two arms and two faces, the total area transferred from AA to BB is given by

$$A = 4 \times \frac{1}{2} r\theta^2 = 2r^2\theta.$$

The shift in area causes an increase in the capacitance of the $B-N$ capacitor given by

$$\delta C = \frac{\varepsilon_0 A}{d} = \frac{2r^2\theta\varepsilon_0}{d}$$

where d is the distance between the surface of the needle and the top or bottom of the quadrants.

The capacitance of the $A-N$ capacitor decreases by the same amount.

Therefore, the increase in the energy of the $B-N$ capacitor

$$= \frac{1}{2} \times \delta C \times (\text{P.D.})^2$$

$$= \frac{1}{2} \times \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_b)^2 = \frac{r^2\theta\varepsilon_0}{d}(V_n - V_b)^2.$$

Similarly, the decrease in the energy of the $A-N$ capacitor

$$= \frac{r^2\theta\varepsilon_0}{d}(V_n - V_a)^2.$$

$\therefore$ net increase in the energy of the system

$$= \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2].$$

In addition to this net gain in electrical potential energy, work has also to be done in twisting the suspension fibre by an angle θ. It is given by $\frac{1}{2}c\theta^2$ where c is the restoring couple per unit twist of the suspension fibre.

$\therefore$ total energy gain

$$= \frac{1}{2}c\theta^2 + \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]. \quad \text{... (i)}$$

As the capacitance of $B-N$ capacitor increases by δC, it draws a charge $\delta C \times (V_n - V_b)$ from the source.

Energy drawn from the source at a constant p.d. $(V_n - V_b)$ is

$$= \text{charge} \times \text{potential difference}$$

$$= \{\delta C \times (V_n - V_b)\} \times (V_n - V_b)$$

$$= \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_b)^2.$$

Similarly, the $A-N$ capacitor, whose capacitance decreases by δC, restores to the source an amount of energy

$$= \frac{2r^2\theta\varepsilon_0}{d} \times (V_n - V_a)^2.$$

$\therefore$ net energy drawn from the source

$$= \frac{2r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]. \quad \text{... (ii)}$$

This must be equal to the total energy gained by the electrometer. Hence equating (*i*) and (*ii*), we get

$$\frac{1}{2}c\theta^2 + \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2] = \frac{2r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]$$

or $$\frac{1}{2}c\theta^2 = \frac{r^2\theta\varepsilon_0}{d}[(V_n - V_b)^2 - (V_n - V_a)^2]$$

$$= \frac{2r^2\theta\varepsilon_0}{d}(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right]$$

or $$\theta = \frac{4r^2\varepsilon_0}{cd}(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right]$$

$$\therefore \quad \theta = k(V_a - V_b)\left[V_n - \frac{V_a + V_b}{2}\right], \quad \text{... (iii)}$$

where $k = \dfrac{4r^2\varepsilon_0}{cd}$

There are two ways of using the electrometer.

(*a*) **Heterostatic use.** The needle is charged to a very high potential which is very large as compared with V_a or V_b. Thus $(V_a + V_b)/2$ is negligible as compared to V_n.

Eq. (*iii*) reduces to $\theta = kV_n(V_a - V_b)$

i.e., $\theta \propto (V_a - V_b)$.

Hence, *the deflection of the needle is directly proportional to the difference of potential between the quadrants AA and BB*. This method is used for the determination of small potential difference only.

(*b*) **Idiostatic use.** The needle voltage V_n is made equal to V_a by connecting the needle to the *AA* quadrants. Smaller potential is given to *BB* pair.

Eq. (*iii*) reduces to $\theta = \frac{k}{2}(V_a - V_b)^2$

So both steady and alternating potential differences can be measured since $(V_a - V_b)^2$ is always positive.

Measurement of ionisation current. The quadrant electrometer used heterostatically is the most suitable instrument to measure ionisation currents which are of the order of 10^{-12} A.

The arrangement consists of an earthed metal chamber *C* containing some gas and two metal plates *P* and *Q* insulated from the chamber (Fig. 5.4). The plate *P* is connected to the positive terminal of a high-tension battery whose negative terminal is earthed. The plate *Q* is connected to one pair of quadrants *AA* of the electrometer. The other pair *BB* is earthed. The needle of the electrometer is charged to a high positive potential.

When the gas in the chamber is ionised, negative ions move toward the plate *P*, the positive ions move toward *Q* and thus an ionisation current is set up. Let *q* be the charge on the plate *Q* at any instant *t*. Then the ionisation current

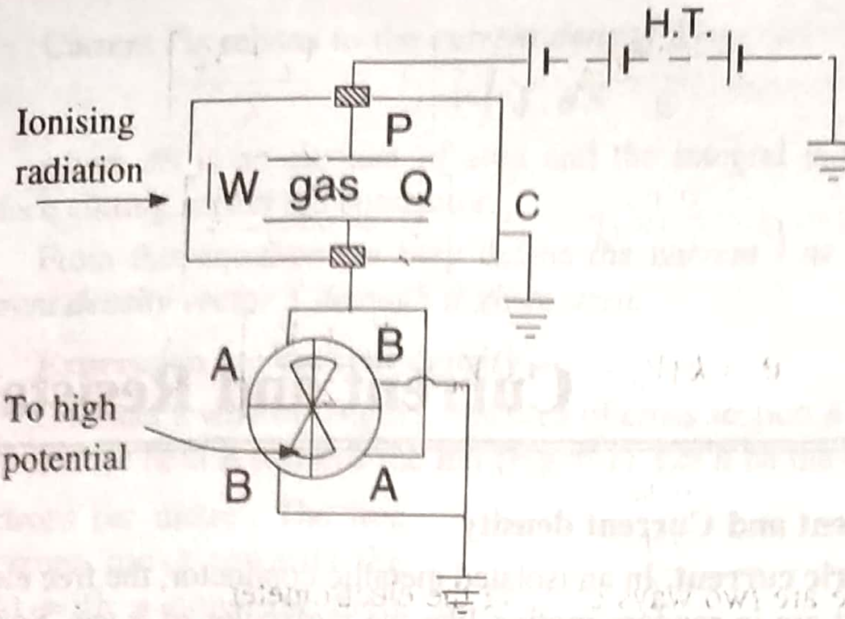


Fig. 5.4

$$i = dq/dt. \quad \text{... (1)}$$

Let C be the capacitance of the electrometer together with the plate Q connected to it and V the potential at the instant t. Then $q = CV$ and Eq. (1) reduces to

$$i = \frac{d}{dt}(CV) = C\frac{dV}{dt}.$$

If θ is the deflection of the needle at any instant,

$$\theta = kV, \text{ where } k \text{ is a constant}$$

or $$V = \frac{\theta}{k}.$$

$$\therefore \quad i = C\frac{dV}{dt} = \frac{C}{k}\left(\frac{d\theta}{dt}\right) \quad \text{... (2)}$$

$d\theta/dt$ is found from the slope of the graph between θ and t. The capacitance C of the electrometer is found from a separate experiment. The constant k is also previously determined by calibrating the electrometer by applying known potential differences across the quadrants, and plotting a graph between θ and V. The slope of this graph gives k. Hence i can be found out.

EXERCISE V

1. Give the theory of the attracted disc electrometer and describe the adjustments to be made before using the instrument. Why is the instrument called an absolute electrometer ?
2. How would you use the attracted disc electrometer to measure (*a*) potential difference between two points and (*b*) the relative permittivity of a dielectric slab.
3. Describe the quadrant electrometer and explain how it may be used to measure (*a*) direct potential (*b*) alternating potential and (*c*) ionisation current.

UNIT II

Carey–Foster Bridge – theory – temperature coefficient of resistance – potentiometer – calibration of ammeter and high range voltmeter – thermoelectricity – laws of thermo e.m.f., intermediate metals, intermediate temperature – measurement of thermo e.m.f. using potentiometer–Peltier effect and Peltier coefficient – Thomson effect and Thomson coefficient – relation between π and σ – thermo electric diagrams and its uses.

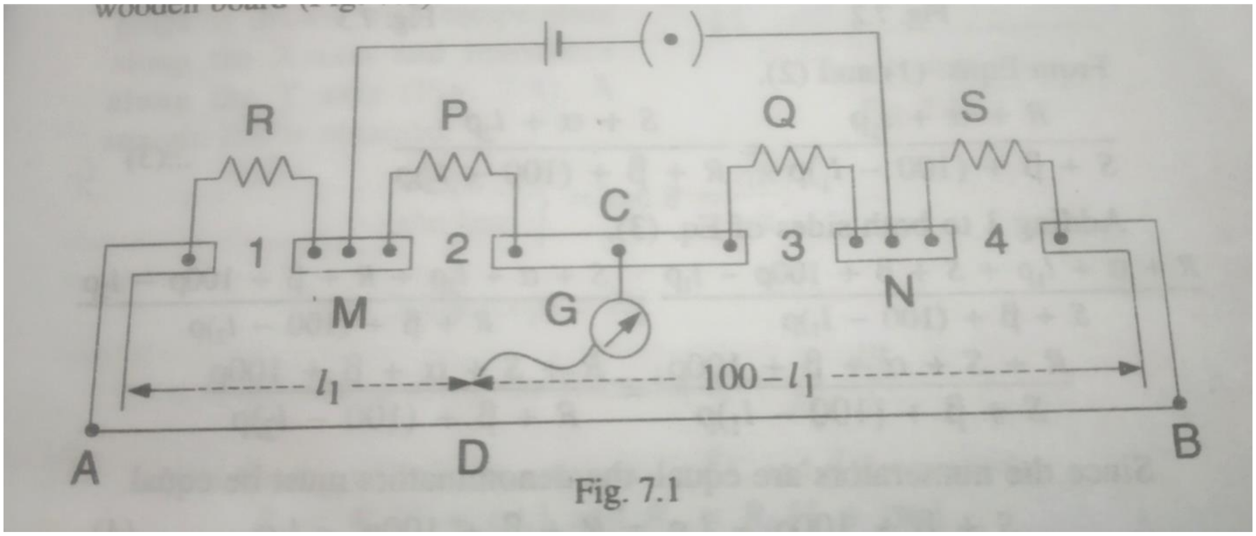
TOPICS:

1. Carey–Foster Bridge – theory – temperature coefficient of resistance –
2. Potentiometer – calibration of ammeter and high range voltmeter
3. Thermoelectricity
   1. Seebeck effect
   2. laws of thermo e.m.f.,
      a. Law of Intermediate Metals
      b. Law of Intermediate Temperature
   3. Measurement of Thermo emf using Potentiometer
   4. Peltier Effect and Peltier Coefficient
   5. Thomson Effect and Thomson coefficient
   6. Thermodynamics of Thermocouple (Relation between π and σ)
   7. Thermo-Electric diagrams
   8. Uses of Thermoelectric diagrams
      i. Determination of Total emf
      ii. Determination of Peltier emf
      iii. Determination of Thomson emf
      iv. Thermoemf in a general couple, neutral temperature and temperature inversion

CAREY FOSTER BRIDGE

DESCRIPTION

1. The carey foster bridge is a form of wheatstone's bridge.
2. It consists of a uniform wire AB of length l metre stretched on a wooden board



Two equal resistances P and Q are connected in gaps 2 and 3. The unknown resistance R is connected in gap 1. A standard resistance S, of the same order of resistance as R, is connected in gap 4. A Lechlanche cell is connected across MN. A galvanometer G is connected between the terminal C and a sliding contact maker D.

THEORY

The contact maker is moved until the bridge is balanced. Let l_1 be the balancing length as measured from end A. let α and β be the end resistances at A and B. Let ρ be the resistance per unit length of the wire. From the principle of wheatstone's bridge,

$$\frac{P}{Q} = \frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho} \text{ ------------1}$$

The resistances R and S are interchanged and the bridge is again balanced. The balancing length l_2 is determined from the same end A.

Then,

$$\frac{P}{Q} = \frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho} \text{ -------------2}$$

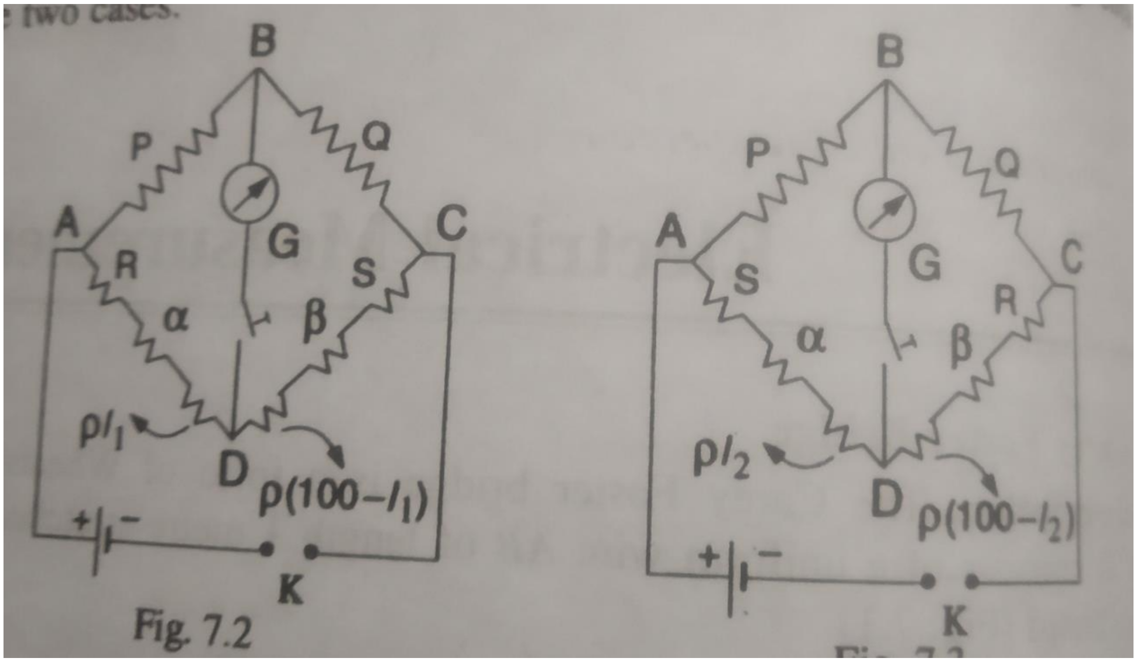


Fig 2 & 3 represents the equivalent wheatstone's bridge circuit in two cases.

From eqns 1 & 2

$$\frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho} = \frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho} \quad \text{---------}3$$

Adding 1 on both sides of eqn 3

$$\frac{R+\alpha+l_1\rho}{S+\beta+(100-l_1)\rho} + 1 = \frac{S+\alpha+l_2\rho}{R+\beta+(100-l_2)\rho} + 1$$

$$\frac{R+\alpha+l_1\rho+S+\beta+100\rho-l_1\rho}{S+\beta+(100-l_1)\rho} = \frac{S+\alpha+l_2\rho+R+\beta+100\rho-l_2\rho}{R+\beta+(100-l_2)\rho}$$

$$\frac{R+\alpha+S+\beta+100\rho}{S+\beta+(100-l_1)\rho} = \frac{S+\alpha+R+\beta+100\rho}{R+\beta+(100-l_2)\rho}$$

Since the numerators are equal the denominators must be equal

$$S+\beta+(100-l_1)\rho = R+\beta+(100-l_2)\rho \quad \text{----------------}4$$

$$S+\beta+100\rho-l_1\rho = R+\beta+100\rho-l_2\rho$$

$$S-l_1\rho = R-l_2\rho$$

$$R = S+(l_1-l_2)\,\rho \quad \text{-------}5$$

To find ρ

A standard resistance of 0.1 Ω is connected in gap 1. A thick copper strip is connected in gap 4. i.e., R = 0.1 Ω and S = 0. The balancing length l_1' is determined. The standard resistance and the thick copper strip are interchanged. The balancing length l_2' is determined.

From eqn 5

$$0.1 = S + \left(l_2' - l_1'\right) \rho$$

or

$$\rho = \frac{0.1}{l_2' - l_1'}$$

Thus by knowing S and ρ, the unknown resistance R is calculated.

Determination of temperature coefficient of resistance

Let R_0 and R_1 be the resistances of a wire at temperatures 0°C and t°C. Then

$$R_t = R_0\,(1 + \alpha t)$$

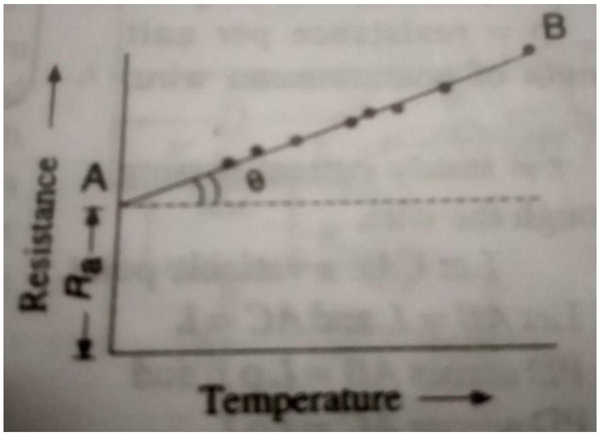
Or

$$\alpha = \frac{R_t - R_0}{R_0\, t}$$

Where α is the temperature coefficient of resistance of the material.

The increase of resistance per unit original resistance per degree rise of temperature is called temperature coefficient of resistance.

The given wire is wound non-inductively in the form of a double spiral on a glass tube. It is immersed in a beaker containing ice at 0°C. The resistance of the wire is determined as above. The resistance of the wire is determined at 10°C, 20°C, 30°C -----100°C. A graph is drawn with temperature along the X – axis and resistance along the Y – axis. A straight line is obtained.



Slope of the line = $tan\,\theta = \frac{dR}{dt}$

Y intercept = R_0

α is calculated using the formula $\alpha = \frac{1}{R_0}\frac{dR}{dt}$

Note:

If $R_{1\,and}\ R_2\ \ be\ the\ resistances\ at\ t_1$℃$and\ t_2$ ℃ $respectively, then$

$$R_1 \ = \ R_0\,[1 + \alpha t_1\,]\ \ and\ \ R_2 \ = \ R_0\,[1 + \alpha t_2\,]$$

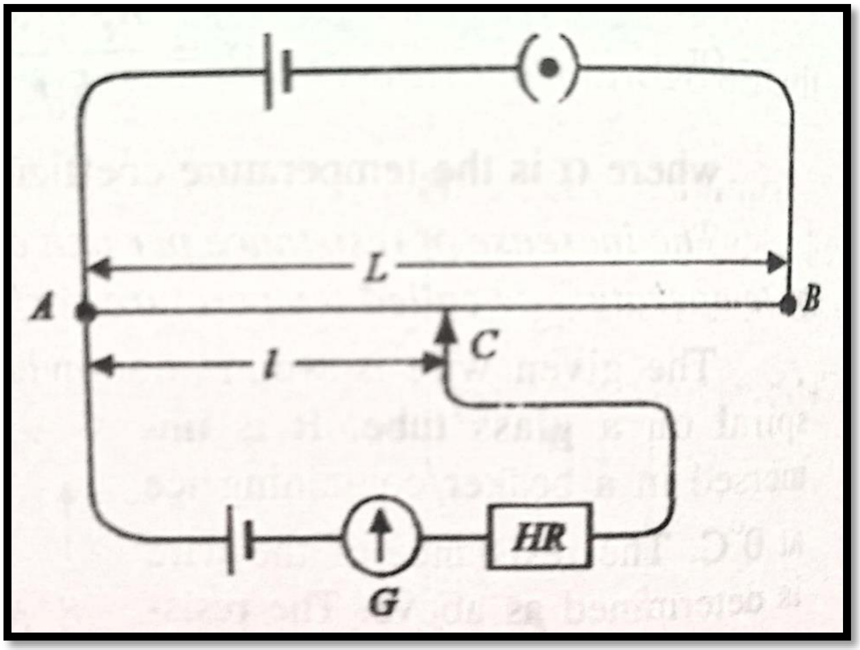
$$\boldsymbol{\alpha = \frac{R_2 - \ R_1}{R_1 t_2 - \ R_2 t_1}}$$

POTENTIOMETER

Principle

A potentiometer is a device for measuring or comparing potential differences.

A potentiometer can be used to measure any electrical quantity which can be converted into a proportionate D.C. potential difference.



It consists of a uniform wire AB of length 10 m stretched on a wooden board. A steady current is passed through the wire AB with the help of a cell of EMF E.

Let ρ = resistance per unit length of potentiometer wire, and

I= steady current passing through the wire.

Consider ‘C’ be a variable point, then AB = L and AC = ℓ

PD across AB = L ρI

PD across AC = ℓ ρI

$$\frac{\text{PD across AB}}{\text{PD across AC}} = \frac{L\rho I}{l\rho I} = \frac{L}{l}$$

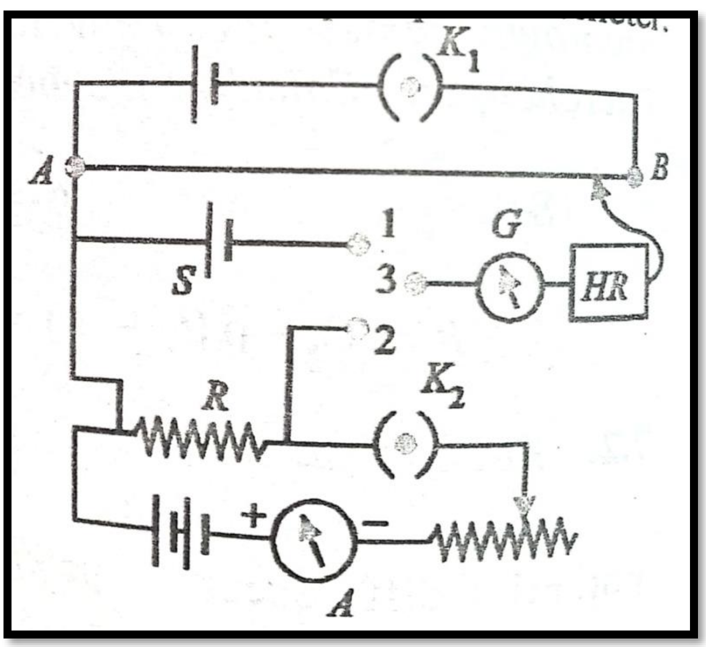
$$\text{PD across AC} = \frac{l}{L}\left(\text{PD across AB}\right)$$

i.e., for a steady current passing through the potentiometer wire AB, the PD across any length is proportional to the length of the wire.

If a D.C. voltmeter is connected between A and the variable point C, it will be noted that the voltmeter registers greater values of PD's as the point C slides from A to B.

CALIBRATION OF AMMETER

Connect the ends of the potentiometer wire to the terminals of a storage cell through a key K_1. S is a standard cell. Connect the ammeter (A) to be calibrated in series with a battery, key K_2, a rheostat and a standard resistance R. When a current I pass through the standard resistance R, the PD across R is IR. This potential drop is measured with the help of potentiometer.



Connect 1 and 3 and balance the EMF of the standard cell against the potentiometer. Find the balancing length (ℓ) from A. the PD per cm of the potentiometer = E/ ℓ.

Connect 2 and 3. Adjust the rheostat so that the ammeter reads a value A_1. Balance the PD across R against the potentiometer and find the balancing length ℓ_1.

PD across R = E ℓ_1 / ℓ

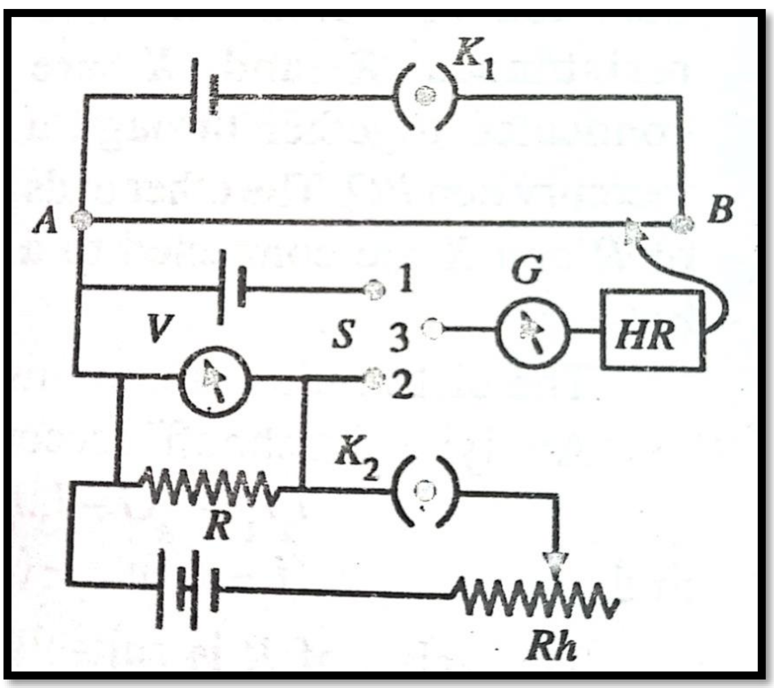
Current through R = E $\ell_1 /$ IR

Correction to ammeter reading (E$\ell_1 /$ IR) – A_1

Similarly, the corrections for other ammeter readings are determined. A Calibration curve is plotted for ammeter, taking ammeter readings on X – axos and corrections on Y – axis.

CALIBRATION OF LOW RANGE VOLTMETER

The connections are made as shoen in fig. the voltmeter is connected parallel to R.



Let ℓ be the balancing length for the standard cell. The PD across R is balanced against the potentiometer. Let ℓ_1 be the balancing length when the voltmeter reads V_1.

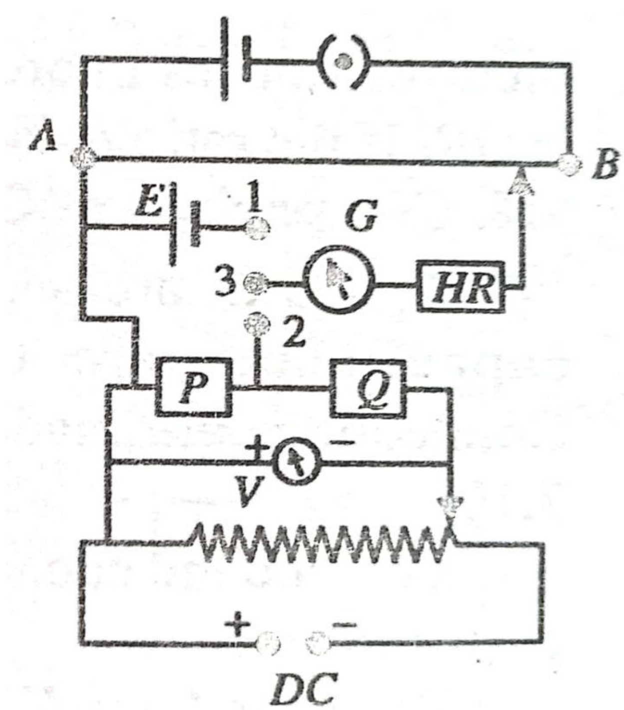
PD across R = $E \ell_1 / \ell$

Correction to the voltmeter = $(E \ell_1 / \ell) - V_1$.

The experiment is repeated for various readings of the voltmeter and a calibration graph is drawn.

CALIBRATION OF HIGH RANGE VOLTMETER

Connections are made as shown in Fig. Take suitable high resistances in P and Q such that the PD across P does not exceed the PD across the potentiometer.



The balancing length ℓ for the standard cell is determined first. Then the PD across P is balanced against the potentiometer and the balancing length ℓ_1 is determined

PD across P = E ℓ_1 / ℓ

PD across P+Q = $\left[\frac{P+Q}{P}\right]\left[\frac{\mathrm{E\,l_1}}{\mathrm{l}}\right]$

Correction to voltmeter = $\left[\frac{P+Q}{P}\right]\left[\frac{\mathrm{E\,l_1}}{\mathrm{l}}\right] - V_1$

The experiment is repeated for various readings of the voltmeter. A calibration curve is plotted for voltmeter, taking voltmeter readings on X – axis and corrections on Y – axis.

= mR. If the ratio m is given, the value of X can be calculated.

7.4. Comparison of Capacitances of Two Capacitors

C_1 and C_2 are two capacitors whose capacitances are to be compared. Connections are made as shown in Fig. 7.10.

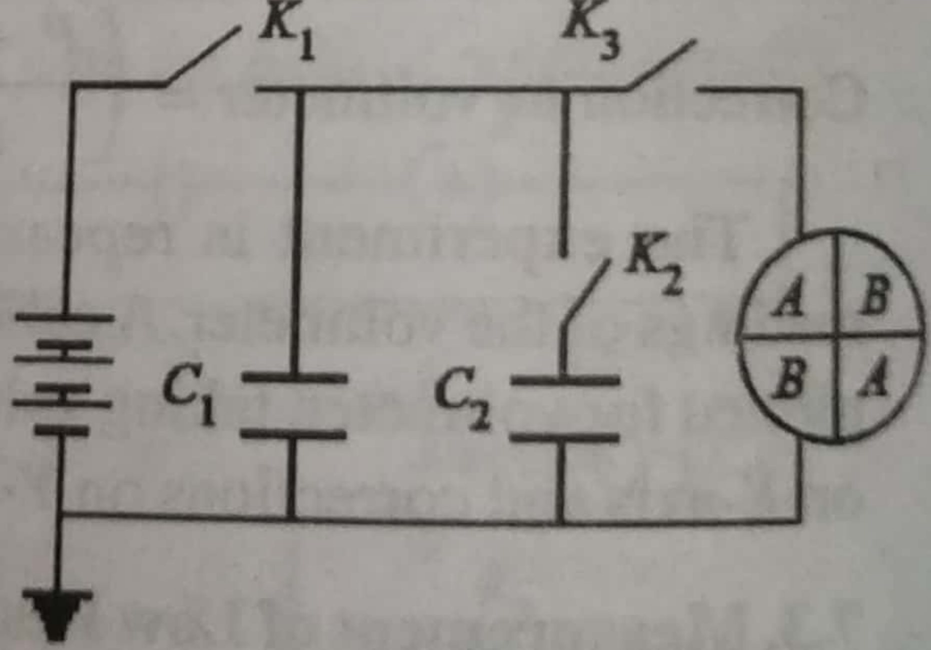


Fig. 7.10.

Let c be the capacitance of quadrant electrometer.

Press the keys K_1 and K_3 and charge the capacitor C_1 to a potential V_1. Let θ_1 be the deflection of the electrometer needle. If Q is the total charge on the capacitor and the electrometer, then

$$Q = (C_1 + c)\, V_1 \qquad \ldots(1)$$

Release the key K_1 and press the key K_2. The charge is now shared by the capacitors C_1, C_2 and c. Let V_2 be the common potential and θ_2 the deflection of the needle.

$$\therefore \quad Q = (C_1 + C_2 + c)\, V_2 \qquad \ldots(2)$$

From (1) and (2), $(C_1 + c)\, V_1 = (C_1 + C_2 + c)\, V_2$

$$\therefore \quad \frac{V_1}{V_2} = \frac{C_1 + C_2 + c}{C_1 + c} \qquad \ldots(3)$$

But, $V_1 = k\theta_1$ and $V_2 = k\theta_2$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\theta_1}{\theta_2} \qquad \ldots(4)$$

From (3) and (4), $\dfrac{C_1 + C_2 + c}{C_1 + c} = \dfrac{\theta_1}{\theta_2}$

$$\therefore \quad 1 + \frac{C_2}{C_1 + c} = \frac{\theta_1}{\theta_2} \quad \text{or} \quad \frac{C_2}{C_1 + c} = \frac{\theta_1 - \theta_2}{\theta_2}$$

The value of c, the capacitance of the electrometer is known.

Hence $\dfrac{C_2}{C_1}$ is calculated.

If the capacitance of the electrometer is negligible,

$$\frac{C_2}{C_1} = \frac{\theta_1 - \theta_2}{\theta_2} \qquad \ldots(5)$$

7.5. Capacitance of a Capacitor (Kelvin's Null Method)

Capacitors C_1, C_2, C_3 and C_4 form the four arms of a Wheatstone's bridge (Fig. 7.11). The values of C_1 and C_2 are known. C_3 is a variable capacitor whose capacitance is *also known*. The capacitance of C_4 is to be determined. A quadrant electrometer used heterostatically with the needle charged to a high potential has its A quadrants connected to the point B and the B quadrants to the point D. A battery is connected between A and C.

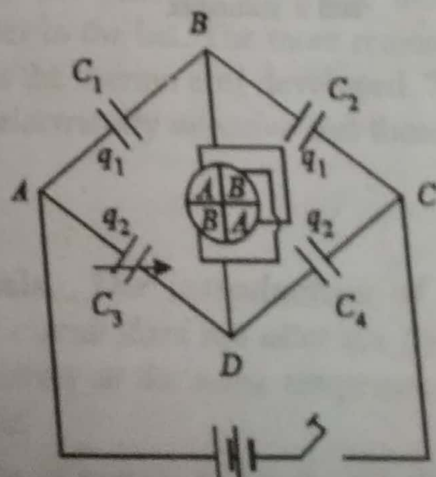


Fig. 7.11.

C_3 is adjusted for null deflection in the electrometer. Now points B and D are at the same potential *i.e.*, $V_B = V_D$

$$\therefore \quad V_A - V_B = V_A - V_D \qquad \ldots(1)$$

and $$V_B - V_C = V_D - V_C \quad ...(2)$$

Capacitors C_1 and C_2 have equal charge, say q_1, and C_3 and C_4 have charge q_2.

$$\therefore \quad V_A - V_B = \frac{q_1}{C_1},\ V_A - V_D = \frac{q_2}{C_3} \quad ...(3)$$

$$V_B - V_C = \frac{q_1}{C_2},\ V_D - V_C = \frac{q_2}{C_4} \quad ...(4)$$

From Eqs. (1), (2) and (3),

$$\frac{q_1}{C_1} = \frac{q_2}{C_3} \quad \text{or} \quad \frac{q_1}{q_2} = \frac{C_1}{C_3} \quad ...(5)$$

From Eqs. (2) and (4),

$$\frac{q_1}{C_2} = \frac{q_2}{C_4} \quad \text{or} \quad \frac{q_1}{q_2} = \frac{C_2}{C_4} \quad ...(6)$$

From Eqs. (5) and (6),

$$\frac{C_1}{C_3} = \frac{C_2}{C_4}$$

$$\therefore \quad C_4 = C_3 \frac{C_2}{C_1}$$

Thus, knowing the values of C_1, C_2 and C_3, the value of C_4 can be calculated.

EXERCISE VII

1. Explain with necessary theory how a Carey Foster bridge may be used to compare two nearly equal resistances. Hence show how the specific resistance of the material of the wire can be determined.
2. Define "temperature coefficient of resistance". How is it determined using the Carey-Foster Bridge ?
3. Explain the theory of potentiometer. How will you use it to calibrate an ammeter and a voltmeter.

8

Thermo-electricity

8.1. Seebeck Effect.

When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed in the circuit (Fig. 8.1). This causes a current to flow in the circuit as indicated by the deflection in the galvanometer *G*. This phenomenon is called the *Seebeck effect.* This arrangement is called a *thermocouple*. The emf developed is called *thermo emf.* The thermo emf so developed depends on the temperature difference between the two junctions and the metals chosen for the couple. Seebeck arranged the metals in a series as follows :

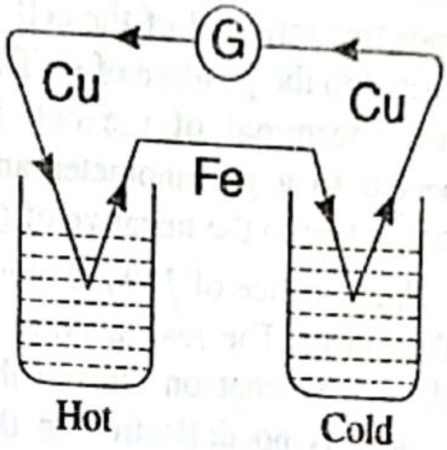


Fig. 8.1

Bi, Ni, Pd, Pt, Cu, Mn, Hg, Pb, Sn, Au, Ag, Zn, Cd, Fe, Sb.

When a thermocouple is formed between any two of them, the thermoelectric current flows through the hot junction from the metal occurring earlier to the metal occurring later in the list. The more removed are the two metals in the list, the greater is the thermo emf developed. The metals to the left of Pb are called *thermoelectrically negative* and those to its right are *thermoelectrically positive.*

8.2. Laws of thermo e.m.f.

(*i*) **Law of Intermediate Metals.** *The introduction of any additional metal into any thermoelectric circuit does not alter the thermo emf provided the metal introduced is entirely at the same temperature as the point at which the metal is introduced.*

If ${}_aE_b$ is the emf for a couple made of metals *A* and *B*, and ${}_bE_c$ that for the couple of metals *B* and *C*, then the emf for couple of metals *A* and *C* is given by

$${}_aE_c = {}_aE_b + {}_bE_c$$

(ii) **Law of Intermediate Temperatures.** *The thermo emf E_1^3 of a thermocouple whose junctions are maintained at temperatures T_1 and T_3 is equal to the sum of the emfs E_1^2 and E_2^3 when the junctions are maintained at temperatures T_1, T_2 and T_2, T_3 respectively.* Thus

$$E_1^3 = E_1^2 + E_2^3$$

8.3. Measurement of Thermo EMF using Potentiometer

Thermo emfs are very small, of the order of only a few millivolts. Such small emfs are measured using a potentiometer. A ten-wire potentiometer of resistance R is connected in series with an accumulator and resistance boxes P and Q (Fig. 8.2). A standard cell of emf E is connected in the secondary circuit. The positive terminal of the cell is connected to the positive of Q. The negative terminal of the cell is connected to a galvanometer and through a key to the negative of Q.

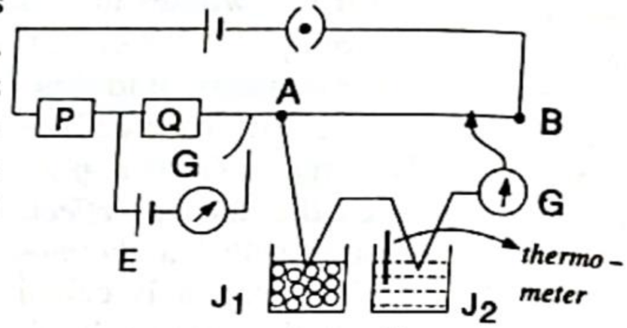


Fig. 8.2

A resistance of *100 ER* ohms is taken in Q. The resistance in P is adjusted so that on closing the key, there is no deflection in the galvanometer. Now, the PD across *100 ER* ohms is equal to E.

$$\left.\begin{array}{l}\text{PD across } R \text{ ohms} \\ \text{of the potentiometer}\end{array}\right\} = \frac{ER}{100\,ER} \text{ volt} = \frac{1}{100} \text{ volt} = 10 \text{ millivolt.}$$

Thus the fall of potential per metre of the potentiometer wire is 1 millivolt. So we can measure thermo emf up to 10 millivolt.

Without altering the resistances in P and Q, the positive of the thermocouple is connected to the positive terminal of the potentiometer and the negative of the thermocouple to a galvanometer and jockey. One junction is kept in melting ice and the other junction in an oil bath or in a sand bath. The jockey is moved till a balance is obtained against the small emf e of the thermocouple. Let $AJ = l$ cm be the balancing length. Then,

$$\text{thermo emf } e = \frac{1}{100}\, l \text{ millivolt.}$$

Keeping the cold junction at 0°C, the hot junction is heated to different temperatures. The thermo emf generated is determined for different temperatures of the hot junction. A graph is drawn between thermo emf and the temperature of the hot junction (Fig. 8.3). The graph is a parabolic curve.

The thermo emf E varies with temperature according to

$E = at + bt^2$, where a and b are constants. The thermo emf increases as the temperature of the hot junction increases, reaches a maximum value T_n, then decreases to zero at a parti-cular temperature T_i. On further increasing the difference of temperature, emf is reversed in direction.

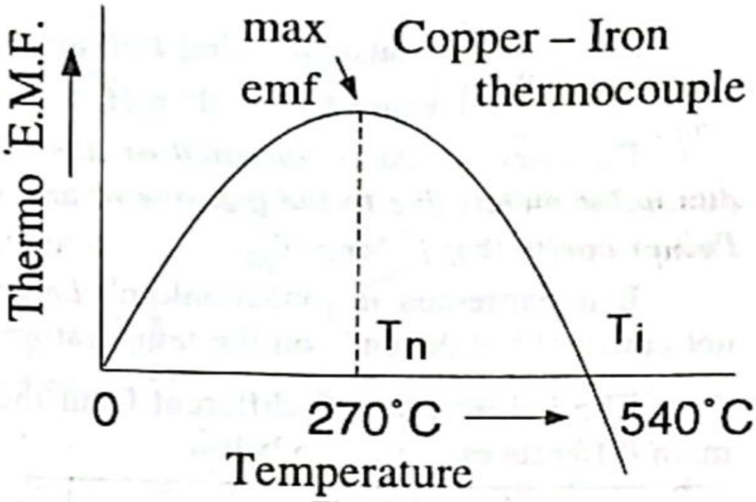


Fig. 8.3

For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes maximum is called the neutral temperature (T_n) *for the given thermocouple.*

For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes zero and changes its direction is called the inversion temperature (T_i) *for the given thermocouple.*

T_n is a constant for the pair of metals. T_i is variable. T_i is as much above the neutral temperature as the cold junction is below it.

8.4. Peltier Effect

Consider a copper-iron thermocouple (Fig. 8.4). When a current is allowed to pass through the thermocouple in the direction of arrows (from A to B), heat is absorbed at the junction B and generated at the junction A. This absorption or evolution of heat at a junction when a current is sent through a thermocouple is called Peltier effect. The Peltier effect is a reversible phenomenon. If the direction of the current is reversed, then there will be cooling at the junction A and heating at the junction B.

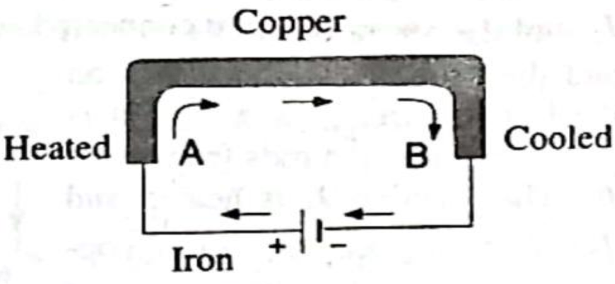


Fig. 8.4

When an electric current is passed through a closed circuit made up of two different metals, one junction is heated and the other junction is cooled. This is known as Peltier effect.

The amount of heat H absorbed or evolved at a junction is proportional to the charge q passing through the junction. *i.e.*,

$$H \propto q \quad \text{or} \quad H \propto It$$

or
$$H = \pi It$$

where π is a constant called *Peltier coefficient.*

When $I = 1A$ and $t = 1s$, then $H = \pi$.

The energy that is liberated or absorbed at a junction between two dissimilar metals due to the passage of unit quantity of electricity is called Peltier coefficient.

It is expressed in joule/coulomb *i.e.*, volt. The Peltier coefficient is not constant but depends on the temperature of the junction.

The Peltier effect is different from the I^2R Joule heating effect. The main differences are given below.

	Peltier Effect	*Joule Effect*
1.	It is a reversible effect.	It is an irreversible effect.
2.	It takes place at the junctions only.	It is observed throughout the conductor.
3.	It may be a heating or a cooling effect.	It is always a heating effect.
4.	Peltier effect is directly proportional to I ($H = \pm \pi It$)	Amount of heat evolved is directly proportional to the square of the current.
5.	It depends upon the direction of the current.	It is independent of the direction of the current.

Demonstration of Peltier effect – S.G. Starling Method.

Fig. 8.5 shows a bismuth bar between two bars of antimony. Two coils C_1 and C_2 of insulated copper wire are wound over the two junctions J_1 and J_2. These coils are connected across the two gaps of a metre-bridge and the balance-point is found on the bridge wire. Now a current is passed through the rods from *Sb* to *Bi*. The junction J_1 is heated and J_2 is cooled. The resistance of copper varies rapidly with change of temperature. Hence the balance in the bridge is immediately upset. The galvanometer shows a deflection. If the current is reversed, the deflection in the galvanometer also gets reversed. This shows that the junction J_1 is now cooled and J_2 is heated.

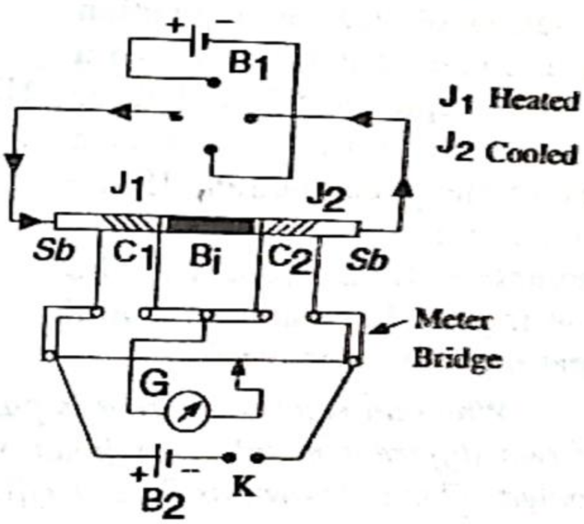


Fig. 8.5

8.5. Thomson Effect

Consider a copper bar *AB* heated in the middle at the point *C* (Fig. 8.6). A current is passed from *A* to *B*. It is observed that heat is absorbed

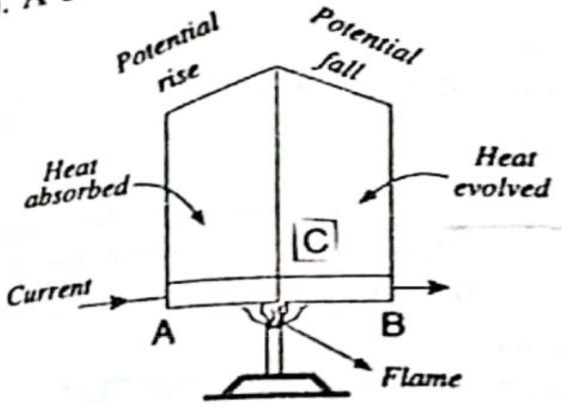


Fig. 8.6

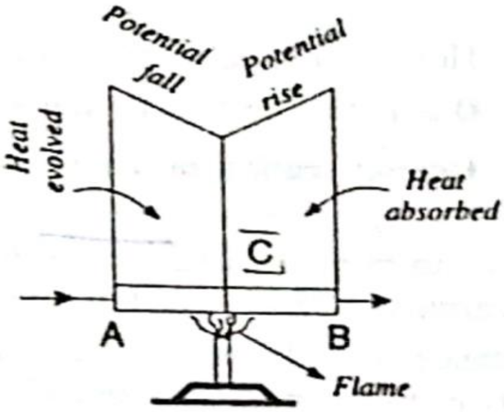


Fig. 8.7

in the part *AC* and evolved in the part *CB*. This is known as *Positive Thomson effect*. Similar effect is observed in metals like Ag, Zn, Sb and Cd.

In the case of an iron bar *AB*, heat is evolved in the part *AC* and absorbed in the part *CB* (Fig. 8.7). This is known as *Negative Thomson effect*. Similar effect is observed in metals like Pt, Ni, Co and Bi.

For lead, the Thomson effect is zero.

The Thomson effect is reversible.

In the case of copper, the hotter parts are at a higher potential than the colder ones. It is opposite in the case of iron. Heat is either absorbed or evolved when current passes between two points having a difference of potential. Therefore, the passage of electric current through a metal having temperature gradient results in an absorption or evolution of heat in the body of the metal.

When a current flows through an unequally heated metal, there is an absorption or evolution of heat throughout in the body of the metal. This is known as 'Thomson effect'.

Thomson Coefficient. *The Thomson coefficient σ of a metal is defined as the amount of heat energy absorbed or evolved when a charge of 1 coulomb flows in the metal between two points which differ in temperature by $1^\circ C$.*

Thus, if a charge of q coulomb flows in a metal between two points having a temperature difference of 1°C, then

heat energy absorbed or evolved = σq joule.

But if E volt be the Thomson emf developed between these points then this energy must be equal to Eq joule.

$$\therefore \quad \sigma q = Eq$$

or

$$\sigma = E.$$

Thus *the Thomson coefficient of a metal, expressed in joule per coulomb per °C, is numerically equal to the emf in volt, developed between two points differing in temperature by 1°C.*

Hence it may also be expressed in volt per °C.

σ is not a constant for a given metal. It is a function of temperature.

Demonstration of Thomson effect.

Fig. 8.8. shows Starling's method of demonstrating the Thomson effect. An iron rod *ABC* is bent into *U* shape. Its ends *A* and *C* are dipped in mercury baths. C_1 and C_2 are two insulated copper wires of equal resistance wound round the two arms of the bent rod. C_1 and C_2 are connected in the opposite gaps of a metre bridge. The bridge is balanced. Then the mid-point *B* of the rod is strongly heated. A heavy current is passed through the rod. Then this current will be flowing up the temperature gradient in one arm and down the temperature gradient in the other arm. As a result, one of the coils will be cooled and the other will be warmed. The balance in the bridge will be upset and the galvanometer in its circuit will show a deflection. If the direction of the current is reversed, the deflection in the galvanometer will be reversed.

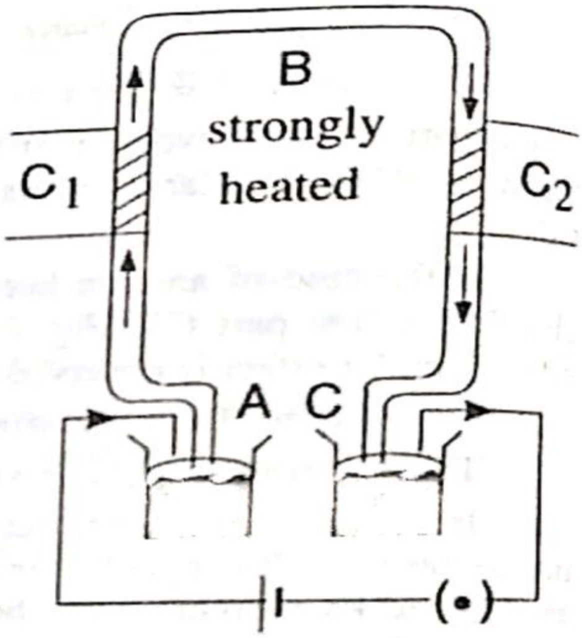


Fig. 8.8

8.6. Thermodynamics of Thermocouple

(Expressions for Peltier and Thomson coefficients.)

Consider a thermocouple consisting of two metals *A* and *B*. Let *T* and $T + dT$ be the temperatures of the cold and hot junctions respectively [Fig. 8.9]. Let π and $\pi + d\pi$ be the Peltier coefficients for the pair at the cold and hot junctions. Let σ_a and σ_b be the Thomson coefficients for the metals *A* and *B* respectively, both taken as positive. When a charge flows through the thermocouple, heat will be absorbed and evolved at the junctions due to Peltier effect and all along the metal due to Thomson effect.

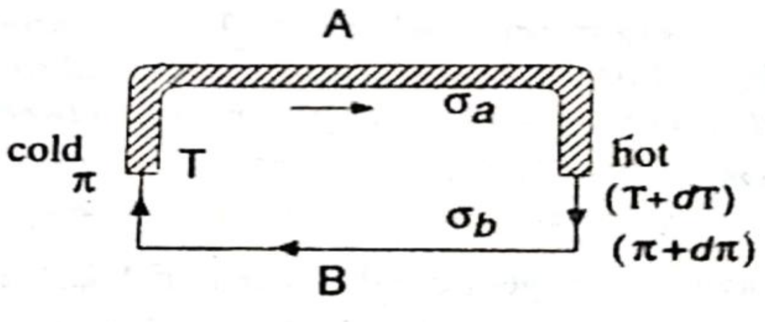


Fig. 8.9

Let 1 *coulomb of charge* flow through the thermocouple in the direction from A to B at the hot junction.

Heat energy absorbed due to Peltier effect at the hot junction = $(\pi + d\pi)$ joules

Heat energy evolved due to Peltier effect at the cold junction = π joules

Heat energy absorbed in the metal A due to Thomson effect

$$= \sigma_a \, dT \text{ joules}$$

Heat energy evolved in the metal B due to Thomson effect

$$= \sigma_b \, dT \text{ joules}$$

∴ Net heat energy absorbed in the thermocouple

$$= (\pi + d\pi - \pi) + (\sigma_a dT - \sigma_b \, dT)$$

$$= d\pi + (\sigma_a - \sigma_b)\, dT$$

This energy is used in establishing a P.D. dE in the thermocouple

$$\therefore \quad dE = d\pi + (\sigma_a - \sigma_b)\, dT \quad \text{... (1)}$$

Since the Peltier and Thomson effects are reversible, the thermocouple acts as a reversible heat engine. Here,

(*i*) the heat energy $(\pi + d\pi)$ joules is absorbed from the source at $(T + dT)$ K and $\sigma_a \, dT$ joule is absorbed in metal A at mean temperature TK.

(*ii*) Also π joule is rejected to sink at T K and $\sigma_b \, dT$ joule is given out in metal B at the mean temperature T K.

Applying Carnot's theorem, we have

$$\frac{\pi + d\pi}{T + dT} + \frac{\sigma_a \, dT}{T} = \frac{\pi}{T} + \frac{\sigma_b \, dT}{T}$$

or
$$\frac{\pi + d\pi}{T + dT} - \frac{\pi}{T} = \frac{(\sigma_b - \sigma_a) dT}{T}$$

or
$$\frac{\pi T + d\pi \, T - \pi T - \pi dT}{T(T + dT)} = \frac{(\sigma_b - \sigma_a) dT}{T}$$

or
$$d\pi . T - \pi . dT = (\sigma_b - \sigma_a)\, dT\, (T + dT)$$

or
$$d\pi . T - \pi dT = (\sigma_b - \sigma_a)\, T dT + (\sigma_b - \sigma_a)\, dT^2$$

or
$$(d\pi . T - \pi dT) = (\sigma_b - \sigma_a)\, T . dT$$

[Neglecting $(\sigma_b - \sigma_a)\, dT^2$]

or
$$T\, [d\pi + (\sigma_a - \sigma_b)\, dT] = \pi \, d\, T$$

But
$$d\pi + (\sigma_a - \sigma_b)\, dT = dE \quad \text{from Eq. (1)}$$

$$\therefore \quad T\, dE = \pi . dT$$

or $$\pi = T.\frac{dE}{dT} \quad ...(2)$$

The quantity (dE/dT) is called the thermoelectric power (P).

Thermoelectric power (P) is defined as the thermo emf per unit difference of temperature between the junctions.

∴ Peltier coefficient = Absolute temperature × thermoelectric power

Differentiating Eq. (2), $\frac{d\pi}{dT} = T\frac{d^2E}{dT^2} + \frac{dE}{dT}$

Substituting the value of (dE/dT) from Eq. (1),

$$\frac{d\pi}{dT} = T\frac{d^2E}{dT^2} + \frac{d\pi}{dT} + (\sigma_a - \sigma_b)$$

or $$(\sigma_a - \sigma_b) = -T.\frac{d^2E}{dT^2}$$

or $$(\sigma_b - \sigma_a) = T.\frac{d^2E}{dT^2} \quad ...(3)$$

If the first metal in the thermocouple is lead, then $\sigma_a = 0$

∴ $$\sigma_b = T.\frac{d^2E}{dT^2} \quad ...(4)$$

Thomson coefficient = absolute temperatureof the cold junction × first derivative of thermoelectric power.

From Eq. (3), $\frac{d^2E}{dT^2} = \frac{(\sigma_b - \sigma_a)}{T}$ or $\frac{d}{dT}\left(\frac{dE}{dT}\right) = \frac{(\sigma_b - \sigma_a)}{T}$

Putting dE/dT from Eq. (2), we have

$$\frac{d}{dT}\left(\frac{\pi}{T}\right) = \frac{(\sigma_b - \sigma_a)}{T}$$

or $$\frac{d}{dT}\left(\frac{\pi}{T}\right) - \left(\frac{\sigma_b - \sigma_a}{T}\right) = 0 \quad ...(5)$$

This gives the relation between Peltier and Thomson's coefficients.

8.7. Thermo-Electric Diagrams

A thermocouple is formed from two metals A and B. The difference of temperature of the junctions is TK. The thermo emf E is given by the equation

$$E = aT + bT^2$$

A graph between E and T is a parabola.

$$\frac{dE}{dT} = a + 2bT$$

dE/dT is called *thermoelectric power.*

A graph between thermoelectric power (dE/dT) and difference of temperature T is a straight line. This graph is called the *thermo-electric power line or the thermo-electric diagram.* Thomson coefficient of lead is zero. So generally thermo-electric lines are drawn with lead as one metal of the thermo-couple. The thermoelectric line of a Cu-Pb couple has a positive slope while that of Fe-Pb couple has a negative slope. Fig. 8.10 shows the power lines for a number of metals.

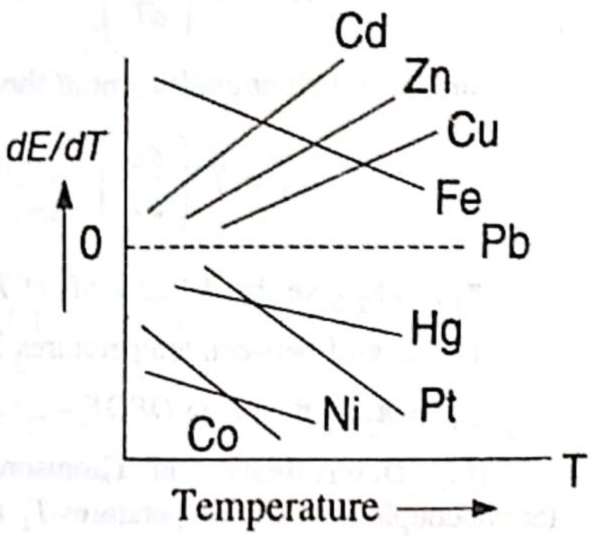


Fig. 8.10

8.8. Uses of Thermoelectric Diagrams

(*i*) **Determination of Total emf.** MN represents the thermo-electric power line of a metal like copper coupled with lead (Fig. 8.11). MN has a positive slope. Let A and B be two points corresponding to temperatures T_1 K and T_2 K respectively along the temperature-axis. Consider a small strip *abdc* of thickness dT with junctions maintained at temperatures T and $(T + dT)$.

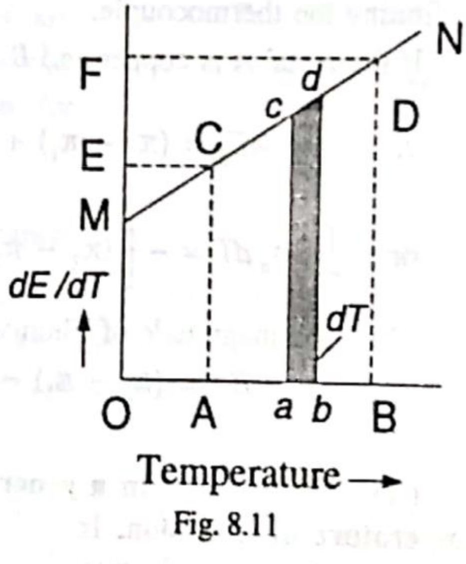


Fig. 8.11

The emf developed when the two junctions of the thermocouple differ by dT is

$$dE = dT\left(\frac{dE}{dT}\right) = \text{area } abdc$$

Total emf developed when the junctions of the couple are at temperatures T_1 and T_2 is

$$E_s = \int_{T_1}^{T_2} dT\left(\frac{dE}{dT}\right) = \text{Area } ABDC$$

(*ii*) **Determination of Peltier emf.** Let π_1 and π_2 be the Peltier coefficients for the junctions of the couple at temperatures T_1 and T_2 respectively.

The Peltier coefficient at the hot junction (T_2) is

$$\pi_2 = T_2\left(\frac{dE}{dT}\right)_{T_2} = OB \times BD = \text{area } OBDF$$

Similarly, Peltier coefficient at the cold junction (T_1) is

$$\pi_1 = T_1\left(\frac{dE}{dT}\right)_{T_1} = OA \times AC = \text{area } OACE$$

π_1 and π_2 give the Peltier emfs at T_1 and T_2 respectively.

Peltier emf between temperatures T_1 and T_2 is

$E_p = \pi_2 - \pi_1 = \text{area } OBDF - \text{area } OACE = \text{area } ABDFECA$

(*iii*) **Determination of Thomson emf.** Total emf developed in thermocouple between temperatures T_1 and T_2 is

$$E_S = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_a - \sigma_b)\, dT$$

Here σ_a and σ_b represent the Thomson coefficients of two metals constituting the thermocouple.

If the metal A is copper and B is lead, then $\sigma_b = 0$.

$$\therefore \quad E_s = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_a\, dT)$$

or $$\int_{T_1}^{T_2} \sigma_a\, dT = -\left[(\pi_2 - \pi_1) - E\right]$$

Thus, the magnitude of Thomson emf is given by

$$E_{th} = (\pi_2 - \pi_1) - E = \text{Area } ABDFECA - \text{Area } ABD\ldots$$
$$= \text{Area } CDFE$$

(*iv*) **Thermo emf in a general couple, neutral temperature and temperature of inversion.** In practice, a thermocouple may consist of any two metals. One of them need not be always lead. Let us consider a thermocouple consisting of any two metals, say *Cu* and *Fe*. *AB* and *CD* are the thermo-electric power lines for *Cu* and *Fe* with respect to lead (Fig. 8.12). Let T_1 and T_2 be

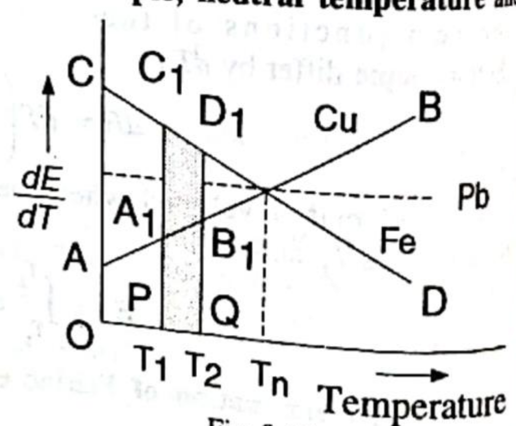


Fig. 8.12

the temperatures of the cold and hot junctions corresponding to points P and Q.

Emf of $Cu - Pb$ thermocouple = Area PQB_1A_1

Emf of $Fe - Pb$ thermocouple = Area PQD_1C_1

$\therefore$ the emf of $Cu - Fe$ thermocouple is

$$E_{Cu}^{Fe} = \text{Area } PQD_1C_1 - \text{Area } PQB_1A_1 = \text{Area } A_1B_1D_1C_1$$

The emf E_{Cu}^{Fe} increases as the temperature of the hot junction is raised and becomes maximum at the temperature T_n, where the two thermoelectric power lines intersect each other. The temperature T_n is called the neutral temperature. As the thermo emf becomes maximum at the neutral temperature, at $T = T_n$, $(dE/dT) = 0$.

Suppose temperatures of the junctions, T_1 and T_2, for a $Cu - Fe$ thermocouple are such that the neutral temperature T_n lies between T_1 and T_2 (Fig. 8.13). Then the thermo emf will be represented by the difference between the areas A_1NC_1 and B_1D_1N because these areas represent opposing emf's. In the particular case when $T_n = (T_1 + T_2)/2$, these areas are equal and the resultant emf is zero. In this case, T_2 is the 'temperature of inversion' for the $Cu - Fe$ thermocouple.

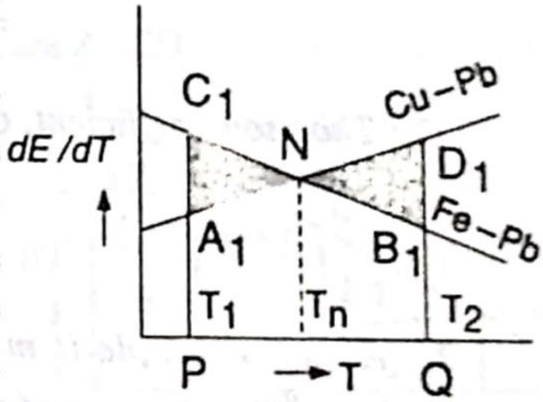


Fig. 8.13

Solved Examples

1. *The emf of a thermocouple, one junction of which is kept at 0°C, is given by $E = at + bt^2$. Determine the neutral temperature, temperature of inversion and the Peltier and Thomson coefficients.*

Sol. $E = at + bt^2$

Now, $t°C = (T - 273)°C$, where T is in absolute degrees.

$$\therefore \quad E = a(T - 273) + b(T - 273)^2.$$

Differentiating, $\dfrac{dE}{dT} = a + 2b(T - 273)$

and $\dfrac{d^2E}{dT^2} = 2b.$

UNIT III

Magnetic induction due to a straight conductor carrying current – magnetic induction on the axis of a solenoid – moving coil ballistic galvanometer – damping correction – determination of absolute capacity of a condenser – self – inductance by Anderson's Bridge method – experimental determination of mutual inductance – coefficient of coupling.

passed through the coil, a deflection of 45° is obtained. Calculate the horizontal component of earth's field.

Sol. $i = \dfrac{5\sqrt{5}a\,B_H}{8\mu_0 N}\tan\theta$

$$B_H = \frac{8\mu_0 N i}{5\sqrt{5}a\tan\theta} = \frac{8\,(4\pi\times10^{-7})\times50\times0.1}{5\sqrt{5}\times0.2\times\tan45^\circ} = 22.48\times10^{-6}\,T.$$

10.6. Magnetic Induction at any point on the axis of a Solenoid

Let L represent the length of the solenoid and N the total number of turns in its winding (Fig. 10.12). The number of turns per unit length is then N/L. a is the radius of the solenoid. A current i is flowing in the solenoid. The solenoid contains air in its core. Let us find the magnetic induction B at a point P on the axis of the solenoid.

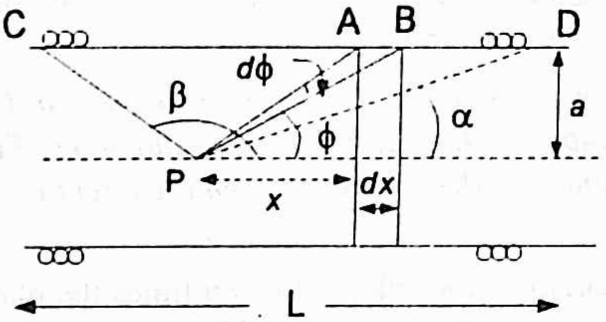


Fig. 10.12

Consider an elementary length dx of the solenoid, at a distance x from P. We can regard this element AB as a circular coil of radius a containing $N\,dx/L$ turns.

Magnetic induction at P due to the element dx is

$$dB = \frac{\mu_0 i a^2}{2}\cdot\frac{N dx}{L}\cdot\frac{1}{(a^2+x^2)^{3/2}} \qquad \text{... (1)}$$

Let us use the angle ϕ instead of x as the independent variable. Then,

$x = a\cot\phi\;;\; dx = -\,a\,\mathrm{cosec}^2\phi\,d\phi$

Substituting these values of x and dx in Eq. (1), we get

$$dB = -\frac{\mu_0 i a^2}{2}\cdot\frac{N}{L}\cdot\frac{a\,\mathrm{cosec}^2\phi\,d\phi}{[a^2+a^2\cot^2\phi]^{3/2}} = -\frac{\mu_0 iN}{2L}\sin\phi\,d\phi$$

The magnetic induction at P due to the entire length of the solenoid

$$B = -\frac{\mu_0 iN}{2L}\int_\beta^\alpha \sin\phi\,d\phi$$

$$= \frac{\mu_0 iN}{2L}[\cos\alpha - \cos\beta] \qquad \text{... (2)}$$

The direction of **B** is parallel to the axis of the solenoid.

Note. If the core of the solenoid consists of magnetic material of permeability μ, the magnetic induction inside such a solenoid is

$$B = \frac{\mu iN}{2L}[\cos\alpha - \cos\beta]$$

where $\mu = \mu_0 \mu_r$ and $\mu = B/H$.

Special cases

(*i*) At a point well inside a very long solenoid : $\alpha = 0, \beta = 180°$.

$$\therefore \quad B = \mu_0 iN/L \quad \text{... (3)}$$

(*ii*) At an axial point at one end of a long solenoid : $\alpha = 0,\ \beta = 90°$.

$$B = \mu_0 iN/2L \quad \text{... (4)}$$

Hence the magnetic induction at either end is one-half its magnitude at points well inside the solenoid.

Example 1. *A solenoid of 1200 turns is wound uniformly in a single layer on a glass tube 2m long and 0.2 m in diameter. Find the magnetic induction at the centre of the solenoid, when a current of 2A flows through it.*

Sol. The length of the solenoid is ten times the diameter so that the formula for a long solenoid can be used. Here,

$$N = 1200,\ L = 2m,\ i = 2A,\ \mu_0 = 4\pi \times 10^{-7}\ Wb\,A^{-1}\,m^{-1}.$$

$$\therefore B = \frac{\mu_0 iN}{L} = \frac{(4\pi \times 10^{-7}) \times 2 \times 1200}{2} = 1.51 \times 10^{-3}\ Wb\,m^{-2}.$$

Example 2. *A long solenoid of length 1 m and mean diameter 0.1 m consists of 1000 turns of wire. A current of 20 A flows through it. Calculate the magnetic induction on its axis (i) at its centre, (ii) at one of its ends.*

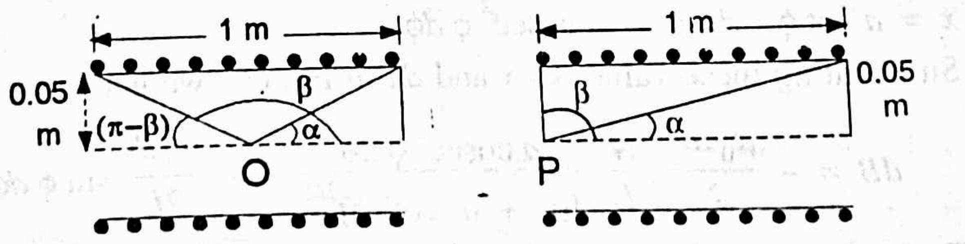


Fig. 10.13

Sol.

(*i*) At the centre O, $\cos\alpha = \dfrac{0.5}{\sqrt{[(0.5)^2 + (0.05)^2]}} = \dfrac{0.5}{\sqrt{(.2525)}}$

$$|\vec{\mu}| = \mu = \text{Area} \times \text{Current} = NiA$$

The direction of $\vec{\mu}$ is given by the right handed screw rule.

$$\therefore \quad \vec{\tau} = \vec{\mu} \times \vec{B} = \mu B \sin\theta \quad \text{... (3)}$$

The *moving coil galvanometer* is based on this concept.

10.11. Moving Coil Ballistic Galvanometer

Principle. *When a current is passed through a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.*

Construction. It consists of a rectangular coil of thin copper wire wound on a non-metallic frame of ivory (Fig. 10.19). It is suspended by means of a phosphor bronze wire between the poles of a powerful horse-shoe magnet. A small circular mirror is attached to the suspension wire. Lower end of the coil is connected to a hair-spring. The upper end of the suspension wire and the lower end of the spring are connected to terminals T_1 and T_2. A cylindrical soft iron core (C) is placed symmetrically inside the coil between the magnetic poles which are also made cylindrical in shape. This iron core concentrates the magnetic field and helps in producing *radial field.*

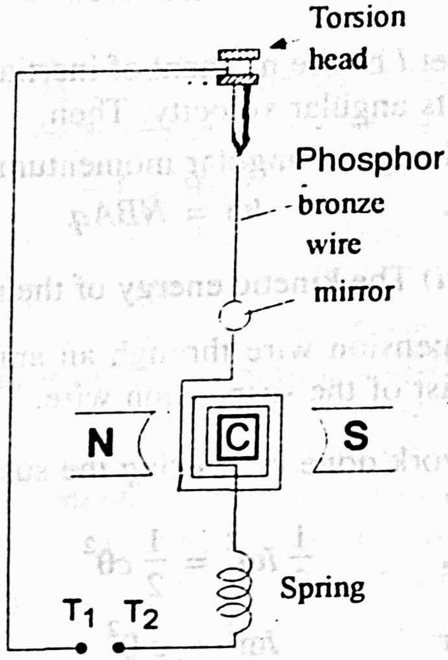


Fig. 10.19

The B.G. is used to measure *electric charge.* The charge has to pass through the coil as quickly as possible and before the coil starts moving. The coil thus gets an impulse and a throw is registered. To achieve this result, a coil of high moment of inertia is used so that the period of oscillation of the coil is fairly large. The oscillations of the coil are practically undamped.

Theory. (*i*) Consider a rectangular coil of N turns placed in a uniform magnetic field of magnetic induction **B** (Fig. 10.20). Let l be the length of the coil and b its breadth.

Area of the coil $= A = lb$.

When a current i passes through the coil,

torque on the coil $= \tau = NiBA$. ... (1)

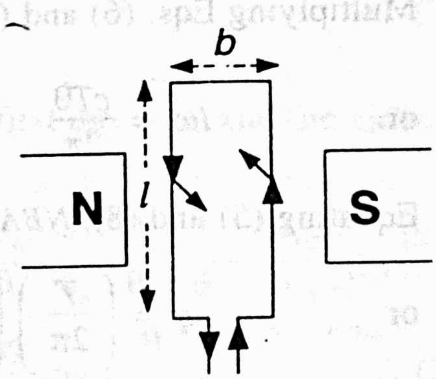


Fig. 10.20

If the current passes for a short interval dt, the angular impulse produced in the coil is

$$\tau\, dt = NiBA\, dt \quad \text{... (2)}$$

If the current passes for t seconds, the total angular impulse given to the coil is

$$\int_0^t \tau\, dt = NBA \int_0^t i\, dt = NBAq \quad \text{... (3)}$$

Here, $\int_0^t i\, dt = q$ = total charge passing through the galvanometer coil.

Let I be the moment of inertia of the coil about the axis of suspension and ω its angular velocity. Then,

change in angular momentum of the coil = $I\omega$ ... (4)

$$\therefore \quad I\omega = NBAq. \quad \text{... (5)}$$

(*ii*) The kinetic energy of the moving system $\frac{1}{2} I\omega^2$ is used in twisting the suspension wire through an angle θ. Let c be the restoring torque per unit twist of the suspension wire. Then,

work done in twisting the suspension wire by an angle $\theta = \frac{1}{2} c\,\theta^2$

$$\therefore \quad \frac{1}{2} I\omega^2 = \frac{1}{2} c\theta^2.$$

$$\text{or} \quad I\omega^2 = c\,\theta^2 \quad \text{... (6)}$$

(*iii*) The period of oscillation of the coil is

$$T = 2\pi \sqrt{\left(\frac{I}{c}\right)} \quad \text{or} \quad T^2 = \frac{4\pi^2 I}{c}$$

$$\therefore \quad I = \frac{T^2 c}{4\pi^2} \quad \text{... (7)}$$

Multiplying Eqs. (6) and (7), $I^2\omega^2 = \frac{c^2 T^2 \theta^2}{4\pi^2}$

$$\text{or} \quad I\omega = \frac{cT\theta}{2\pi} \quad \text{... (8)}$$

Equating (5) and (8), $NBAq = \frac{cT\theta}{2\pi}$

$$\text{or} \quad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta \quad \text{... (9)}$$

This gives the relation between the charge flowing and the ballistic throw θ of the galvanometer. $q \propto \theta$.

$\left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)$ is called the *ballistic reduction factor (K)*.

$$\therefore \quad q = K\,\theta \quad \text{... (10)}$$

Correction for Damping in Ballistic Galvanometer

We have assumed that the whole of the kinetic energy imparted to the coil is used in twisting the suspension of the coil. In actual practice, the motion of the coil is damped by air resistance and the induced current produced in the coil. The first throw of the galvanometer is, therefore, smaller than it would have been in the absence of damping. The correct value of first throw is however obtained by applying damping correction.

Let $\theta_1, \theta_2, \theta_3, \ldots$ be the successive maximum deflections from zero position to the right and left (Fig. 10.21). Then it is found that

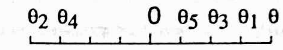


Fig. 10.21

$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = d \quad \text{... (1)}$$

The constant d is called the *decrement* per half vibration.

Let $d = e^{\lambda}$ so that $\lambda = \log_e d$

Here λ is called the *logarithmic decrement.*

For a complete vibration,

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}$$

Let θ be the true *first* throw in the absence of damping.

$\theta > \theta_1$. The first throw θ_1 is observed after the coil completes a quarter of vibration. In this case, the value of the decrement would be $e^{\lambda/2}$.

$$\therefore \quad \frac{\theta}{\theta_1} = e^{\lambda/2} \approx \left(1 + \frac{\lambda}{2}\right)$$

$$\text{or} \quad \theta = \theta_1\left[1 + \frac{\lambda}{2}\right] \quad \text{... (2)}$$

We can calculate λ by observing the first throw θ_1 and the eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2} \cdot \frac{\theta_2}{\theta_3} \cdot \frac{\theta_3}{\theta_4} \cdot \frac{\theta_4}{\theta_5} \cdot \frac{\theta_5}{\theta_6} \cdot \frac{\theta_6}{\theta_7} \cdot \frac{\theta_7}{\theta_8} \cdot \frac{\theta_8}{\theta_9} \cdot \frac{\theta_9}{\theta_{10}} \cdot \frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

$$\text{or} \quad \lambda = \frac{1}{10} \log_e \frac{\theta_1}{\theta_{11}} = \frac{2.3026}{10} \log_{10} \frac{\theta_1}{\theta_{11}}. \quad \text{... (3)}$$

$$\therefore \qquad q = \left(\frac{T}{2\pi}\right)\left(\frac{c}{NBA}\right)\theta_1\left(1+\frac{\lambda}{2}\right) \qquad \dots (4)$$

Dead-beat and Ballistic galvanometers

Galvanometers are classified as (*i*) dead-beat or aperiodic and (*ii*) ballistic galvanometers.

A moving coil galvanometer in which the coil is wound on a metallic conducting frame is known as a dead-beat galvanometer. It is called "dead-beat" because it gives a steady deflection without producing any oscillation, when a steady current is passed through the coil.

Conditions for a moving coil galvanometer to be dead beat :

(*i*) *Moment of inertia of the system should be small.*

(*ii*) *Coil should be mounted on a conducting frame.*

(*iii*) *Suspension fibre should be comparatively thicker.*

Conditions for a moving coil galvanometer to be ballistic :

(*i*) *The moment of inertia of moving system should be large.*

(*ii*) *Suspension fibre should be very fine.*

(*iii*) *Air resistance should be small.*

(*iv*) *The damping should be small i.e., the coil should be wound on a non-conducting frame.*

10.12. Current and Voltage Sensitivities of a moving-coil galvanometer.

The figure of merit or current sensitivity (S_c) of a moving coil mirror galvanometer is the current that is required to produce a deflection of 1 mm on a scale kept at a distance of 1 metre from the mirror.

It is expressed in μA/mm.

The voltage sensitivity (S_v) is the p.d. that should be applied to the galvanometer to produce a deflection of 1 mm on a scale at a distance of 1 metre.

It is expressed in μV/mm.

To determine the current and voltage sensitivities of a galvanometer, the circuit shown in Fig. 10.22 is used. Two resistance boxes *P* and *Q* and a key *K* are connected in series with a lead accumulator of emf *E*. Between the ends of *P*, a resistance box *R* and the M.G., through a commutator, are connected.

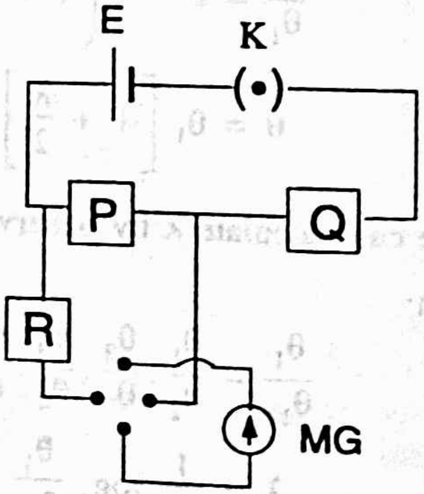


Fig. 10.22

$$\frac{L}{r} = \frac{T}{2\pi}\frac{\theta_1}{\phi}\left(1 + \frac{\lambda}{2}\right)$$

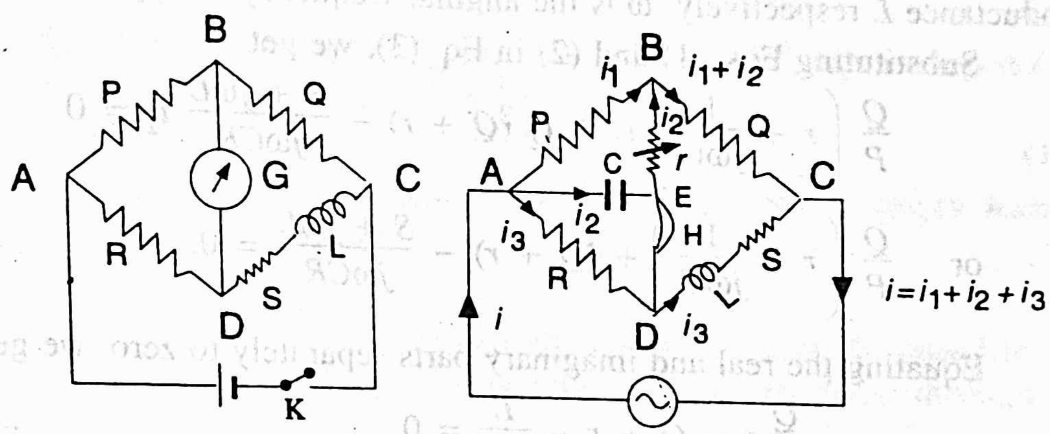
or
$$L = \frac{rT}{2\pi}\frac{\theta_1}{\phi}\left(1 + \frac{\lambda}{2}\right)$$

11.6. Determination of Self-inductance by Anderson's Bridge Method

The experiment is performed in two stages.

(*a*) *D.C. balance.* The circuit connections are shown in Fig. 11.7 (*a*). The given coil of self-inductance *L* and resistance *S* is connected in arm *DC*. The ratio arms *P* and *Q* are fixed to ratio 1 : 1. The resistance *R* is adjusted for balance. This gives the approximate value of resistance *S* of the coil. The experiment is repeated by making the *P* : *Q* ratio to be 10 : 1 and 100 : 1. The accurate value of D.C. resistance of the coil is found by the Wheatstone's bridge relation,

$$\frac{P}{Q} = \frac{R}{S} \quad \text{or} \quad S = R\frac{Q}{P}.$$



(*a*) (*b*)

Fig. 11.7

(*b*) *A.C. balance.* An ac source (oscillator) is connected between *A* and *C* (Fig. 11.7 b). A variable non-inductive resistance *r* is connected in series with a capacitance *C* and this combination is connected in parallel with the arm *AB*. A headphone *H* is connected between *E* and *D*. The resistance *r* is adjusted until minimum sound is heard in the headphone. The value of *L* is calculated using the formula

$$L = C\,[RQ + r\,(R + S)]$$

Theory. Let the instantaneous currents in the different arms be as shown in Fig. 11.7 *b*.

At the time of balance (*i.e.*, no current through headphone),

potential at *E* = potential at *D*.

Applying Kirchoff's II law, we have

(*i*) for mesh *ABEA*,

$$i_1 P - i_2\left(r + \frac{1}{j\omega C}\right) = 0$$

or $$i_1 = \frac{1}{P}\left(r + \frac{1}{j\omega C}\right) i_2 \quad \text{... (1)}$$

(*ii*) for mesh *AEDA*,

$$\frac{i_2}{j\omega C} - i_3 R = 0$$

or $$i_3 = \frac{1}{j\omega CR} i_2 \quad \text{... (2)}$$

(*iii*) for mesh *BCDB*,

$$(i_1 + i_2) Q - i_3 (S + j\omega L) + i_2 r = 0$$

or $$i_1 Q + i_2 (Q + r) - i_3 (S + j\omega L) = 0 \quad \text{... (3)}$$

Here $\frac{1}{j\omega C}$ and $j\omega L$ are the impedances offered by capacitor C and inductance L respectively. ω is the angular frequency of applied *a.c.*

Substituting Eqs. (1) and (2) in Eq. (3), we get

$$\frac{Q}{P}\left(r + \frac{1}{j\omega C}\right) i_2 + i_2 (Q + r) - \frac{S + j\omega L}{j\omega CR} i_2 = 0$$

or $$\frac{Q}{P}\left(r + \frac{1}{j\omega C}\right) + (Q + r) - \frac{S + j\omega L}{j\omega CR} = 0. \quad \text{... (4)}$$

Equating the real and imaginary parts separately to zero, we get

$$\frac{Q}{P} r + Q + r - \frac{L}{CR} = 0 \quad \text{... (5)}$$

and $$\frac{Q}{P\omega C} - \frac{S}{\omega CR} = 0 \quad \text{... (6)}$$

Eq. (6) gives $$\frac{P}{Q} = \frac{R}{S} \text{ or } S = R\frac{Q}{P} \quad \text{... (7)}$$

This is condition for D.C. balance.

From Eq. (5), we get

$$\frac{L}{CR} = \frac{Q}{P} r + Q + r$$

or $$L = CR\left[\frac{Q}{P} r + Q + r\right]$$

or $$L = C\left[R\frac{Q}{P} r + RQ + Rr\right]$$

or $$L = C[Sr + RQ + Rr] \quad \text{from Eq. (7)}$$

or $$L = C[RQ + r(R + S)]$$

11.7. Mutual Induction

Consider two coils P and S placed close to each other (Fig. 11.8). When the current flowing in the coil P changes, the magnetic flux linked with the neighbouring coil S also changes. Hence an induced emf is set up in coil S. This phenomenon is called *mutual induction*. The circuit in which the current changes is called the *primary circuit*. The neighbouring circuit in which the emf is induced is called the *secondary circuit*. Any two circuits in which there is mutual induction are known as mutually coupled circuits.

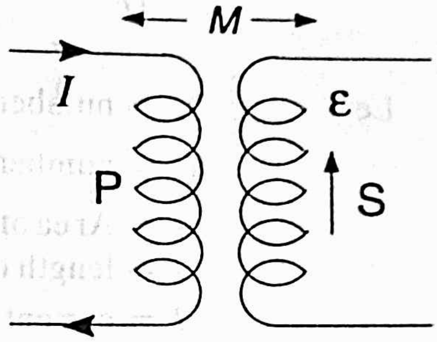


Fig. 11.8

Let I = current in the primary coil P.

The magnetic flux ϕ through the secondary coil S depends upon I.

$$\phi \propto I$$

or $$\phi = MI \quad \text{... (1)}$$

where M is the *coefficient of mutual induction* or simply *mutual inductance* of the two circuits.

If $I = 1$, $\phi = M$.

The mutual inductance of two circuits is numerically equal to the magnetic flux linked with one circuit, when unit current flows through the other.

The emf induced in S is given by

$$\varepsilon = -\frac{d\phi}{dt} = -M\frac{dI}{dt} \quad \text{... (2)}$$

If $dI/dt = 1$, then $\varepsilon = M$ (numerically).

The mutual inductance of two circuits is numerically equal to the e.m.f. in one circuit when the rate of change of current in the other is unity.

In Eq. (2), if ε is in volt and dI/dt in ampere per second, then the mutual inductance is in *henry*. Hence *the mutual inductance between two circuits is 1 henry if a change of current equal to 1 As^{-1} in one circuit induces an emf of 1 volt in the other circuit.*

The mutual inductance depends on the size, shape, number of turns and relative orientation of the two coils. It also depends on the nature of the medium between the two coils.

11.8. Mutual Inductance between two Coaxial Solenoids

Consider a long air-cored solenoid with primary *PP* and secondary *SS* (Fig. 11.9).

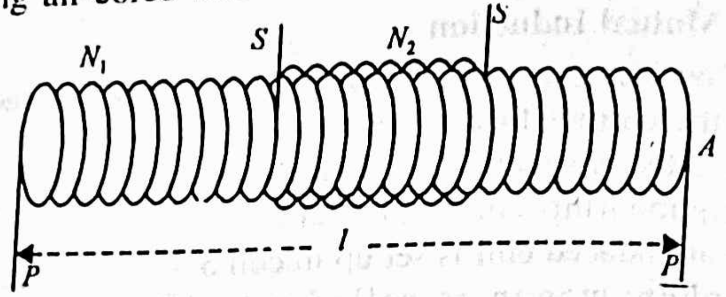


Fig. 11.9

Let, N_1 = number of turns in the primary,

N_2 = number of turns in the secondary,

A = Area of cross section,

l = length of the primary,

I = current in the primary.

$$\left.\begin{array}{l}\text{Magnetic field at any}\\\text{point inside the primary}\end{array}\right\} = B = \frac{\mu_0 N_1 I}{l}$$

∴ magnetic flux through each turn of the primary is

$$BA = \frac{\mu_0 N_1 I A}{l}$$

Since the secondary is wound *closely* over the central portion of the primary, the same flux is also linked with *each turn* of the secondary.

$$\left.\begin{array}{l}\text{Magnetic flux through}\\\text{each turn of the secondary}\end{array}\right\} = \frac{\mu_0 N_1 I A}{l}$$

$$\left.\begin{array}{l}\text{Total magnetic flux through}\\N_2\text{ turns of the secondary}\end{array}\right\} = \phi = \frac{\mu_0 N_1 I A N_2}{l}$$

By definition of mutual inductance,

$$M = \frac{\phi}{I} = \frac{\mu_0 N_1 N_2 A}{l}$$

If the core is a material of permeability μ_r, then

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l} \text{ henry}$$

If there are a number of cores of areas of cross-section A_1, A_2, A_3, etc., and relative permeabilities μ_{r1}, μ_{r2}, μ_{r3}, etc.

$$M = \frac{\mu_0 N_1 N_2}{l} [\mu_{r1} A_1 + \mu_{r2} A_2 + \mu_{r3} A_3 + \ldots] \text{ henry}$$

Example 1. *A solenoid of length 30 cm and area of cross section 10 sq. cm has 1000 turns wound over a core of constant permeability 600. Another coil of 500 turns is wound over the same coil at its middle. Calculate the mutual inductance between them.*

Sol. Here, $N_1 = 1000, N_2 = 500, \mu_r = 600, A = 10\times 10^{-4}\ m^2$, $l = 30\times 10^{-2}$ m.

$$M = \frac{\mu_0\mu_r N_1 N_2 A}{l}$$

$$= \frac{4\pi\times 10^{-7}\times 600\times 1000\times 500\times 10\times 10^{-4}}{30\times 10^{-2}} = 1.257 \text{ henry.}$$

Example 2. *Two coils, a primary of 600 turns and a secondary of 30 turns, are wound on an iron ring of mean radius 0.1 m and cross-section 4×10^{-2}m diameter. Find their mutual inductance (μ_r for iron = 800).*

Sol. $l = 2\pi(0.1) = 0.2\pi$ m, $N_1 = 600$, $N_2 = 30$,

$A = \pi(2\times 10^{-2})^2 = 4\pi\times 10^{-4}\ m^2$.

$$M = \frac{\mu_r\mu_0 N_1 N_2 A}{l}$$

$$= \frac{800\,(4\pi\times 10^{-7})\times 600\times 30\times 4\pi\times 10^{-4}}{0.2\pi} = 3.45\times 10^{-2} \text{ henry}$$

11.9. Experimental determination of mutual inductance

Fig. 11.10 represents the circuit arrangement for the measurement of mutual inductance between two coils P and S. C is a four-segment commutator and r is a very small resistance of the order of 0.01 ohm.

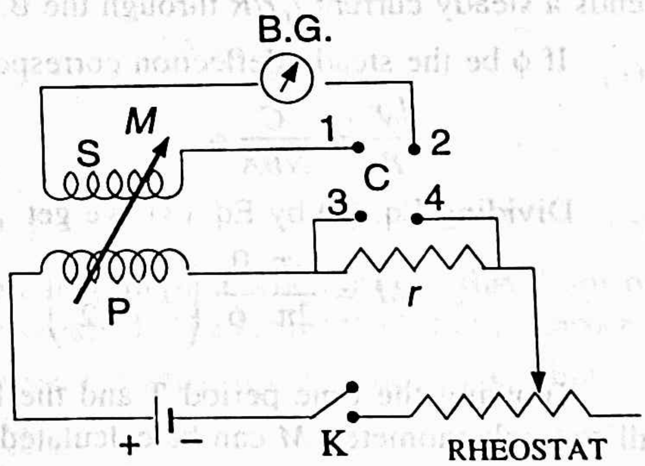


Fig. 11.10

At first, 1 and 2 are connected together so that the secondary circuit is closed through the ballistic galvanometer (B.G.). Now segments 3 and 4 are also connected together to short circuit the resistance r.

When the key K is pressed, the B.G. gives a throw. On pressing the key K, the current in the primary slowly grows. Hence an induced emf is produced in the secondary. Let I be the instantaneous current in the primary.

$$\left.\begin{array}{l}\text{The emf induced}\\ \text{in the secondary}\end{array}\right\} = \varepsilon = -M\frac{dI}{dt}$$

The instantaneous current I' in the secondary is

$$I' = \frac{\varepsilon}{R} = \frac{M}{R}\frac{dI}{dt}. \text{ (numerically)}$$

where R is the total resistance of the secondary circuit.

Hence the total charge passing through the B.G., as the current in the primary grows from zero to a steady maximum value I_0 in time interval t, is

$$q = \int_0^t I' dt = \int_0^t \frac{M}{R} \cdot \frac{dI}{dt} dt = \int_0^{I_0} \frac{M}{R} dI = \frac{MI_0}{R} \quad \text{... (1)}$$

If θ_1 is the first throw in the B.G., due to this charge, then

$$q = \frac{T}{2\pi} \cdot \frac{C}{NBA} \cdot \theta_1 \left(1 + \frac{\lambda}{2}\right)$$

$$\therefore \quad \frac{M}{R} I_0 = \frac{T}{2\pi} \frac{C}{NBA} \theta_1 \left(1 + \frac{\lambda}{2}\right) \quad \text{... (2)}$$

To eliminate I_0 and $C/(NBA)$ from Eq. (2), the contact between 1 and 2, and that between 3 and 4 are broken. The contact between 1 and 3, and that between 2 and 4 are made. The resistance r is now included in the primary circuit. As the value of r is very small, the steady current I_0 in the primary circuit is not altered. The potential difference across r is $I_0 r$. It sends a steady current $I_0 r/R$ through the B.G.

If ϕ be the steady deflection corresponding to this current, then

$$\frac{I_0 r}{R} = \frac{C}{NBA} \phi \quad \text{... (3)}$$

Dividing Eq. (2) by Eq. (3), we get

$$M = \frac{rT}{2\pi} \frac{\theta_1}{\phi} \left(1 + \frac{\lambda}{2}\right)$$

Knowing the time period T and the logarithmic decrement λ of the ballistic galvanometer, M can be calculated.

11.10. Coefficient of coupling

Consider two coils having self-inductances L_1 and L_2 and number of turns N_1 and N_2 (Fig. 11.11). I_1 and I_2 are the currents flowing through the two coils.

Fig. 11.11

Let ϕ_1 and ϕ_2 be the magnetic fluxes linked with each turn of coils 1 and 2 due to their own currents I_1 and I_2 respectively.

The self-inductance of the coils is given by

$$L_1 = \frac{N_1\phi_1}{I_1} \quad \text{... (1)}$$

and

$$L_2 = \frac{N_2\phi_2}{I_2} \quad \text{... (2)}$$

Let ϕ_{12} be the flux per turn in the coil 1 due to current I_2 in coil 2. Similarly, ϕ_{21} is flux per turn linked with coil 2 due to current I_1 in coil 1. Then the mutual inductance between them is given by

$$M = \frac{N_1\phi_{12}}{I_2} = \frac{N_2\phi_{21}}{I_1} \quad \text{... (3)}$$

The whole of the flux from one coil is linked with the other coil. Then

$$\phi_{12} = \phi_2$$

and $\phi_{21} = \phi_1$.

$$\therefore \text{ from Eq. (3), } M = \frac{N_1\phi_2}{I_2} = \frac{N_2\phi_1}{I_1}$$

$$M^2 = \frac{N_1N_2\phi_1\phi_2}{I_1I_2} \quad \text{... (4)}$$

From Eq. (1) and Eq. (2), $L_1L_2 = \dfrac{N_1N_2\phi_1\phi_2}{I_1I_2}$ ... (5)

Hence $M^2 = L_1L_2$

or $M = \sqrt{(L_1L_2)}$ ... (6)

In practice, however, the condition that whole of the flux from one coil links with the other, it not satisfied. The ratio $M/(\sqrt{L_1L_2})$ is known as the *coefficient of coupling* between the coils. It is denoted by k. Thus

$$k = \frac{M}{\sqrt{L_1L_2}} \quad \text{... (7)}$$

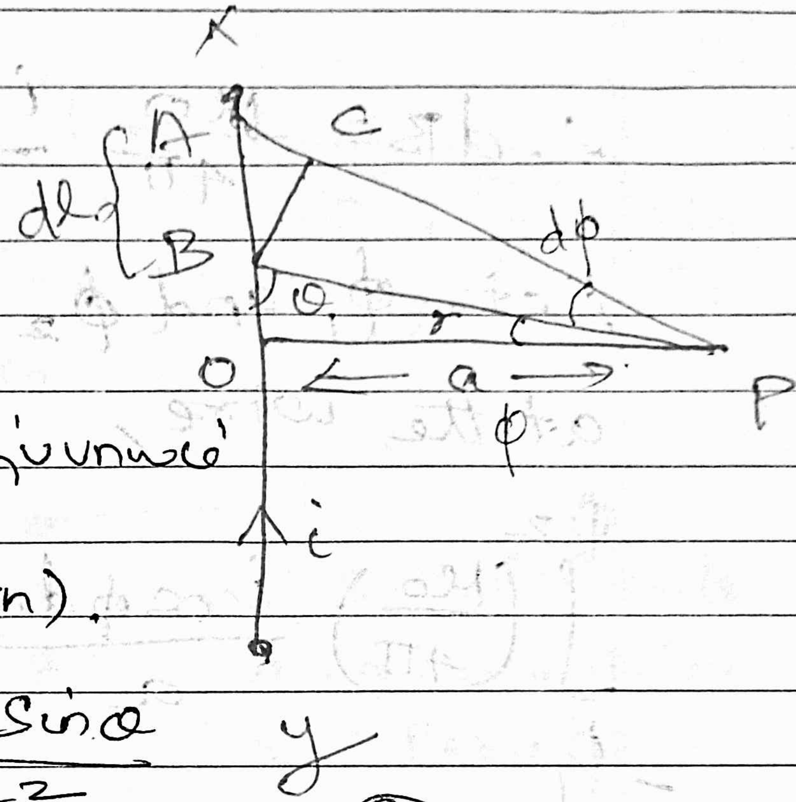
k is a number between 0 and 1, depending upon the geometry of the coils and their relative positions.

If $k = 1$ (maximum value), there is no leakage of flux *i.e., all* the flux produced in one coil is linked with the other and $M = \sqrt{(L_1L_2)}$. This is the maximum possible value of M between the coils of self-inductances L_1 and L_2. If $k = 0$, there is no coupling between the two coils.

11.11. Earth Inductor

Earth inductor is an instrument used for the measurement of magnetic elements of earth. The purpose of an earth inductor is to generate an

2) Magnetic Induction at a point due to a Straight Conductor Carrying Current:-



$\angle OBP = \theta$

$BP = r$

P ல் ஏற்படும் காந்தப்புலம்
due to AB = dB
(mag. Inductn).

$$dB = \frac{\mu_0}{4\pi} \frac{i\,dl\sin\theta}{r^2} \quad \rightarrow (1)$$

from B draw BC ⊥ PA

$\angle OPB = \phi$

$\angle BPA = d\phi$

$BC = dl \sin\theta$
$= r\,d\phi$

$$dB = \frac{\mu_0}{4\pi} \frac{i r\, d\phi}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{i\, d\phi}{r}$$

$\triangle OPB$ $\cos\phi = \frac{a}{r} = r = \frac{a}{\cos\phi}$.

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{i \cos\phi\, d\phi}{a}$$

Let ϕ_1 and ϕ_2 be the angles made at the wire,

$$\int_{-\phi_1}^{\phi_2} \left(\frac{\mu_0}{4\pi}\right) \frac{i \cos\phi\, d\phi}{a} =$$

$$= \frac{\mu_0}{4\pi} \frac{i}{a} \left(\sin\phi\right)_{-\phi_1}^{\phi_2}$$

$\phi_1 = \phi_2 = 90^0$

$$= \frac{\mu_0}{4\pi} \frac{i}{a} \left(\sin\phi_2 + \sin\phi_1 \right)$$

$$\frac{\mu_0}{4\pi} \frac{i}{a} (1+1) = 2$$

$$\frac{\mu_0}{4\pi} \frac{i}{a} \times 2$$

$$\boxed{\frac{\mu_0 i}{2\pi a}.}$$

UNIT IV

Transient current – growth and decay of current in a circuit containing resistance and inductance – growth and decay of charge in a circuit containing resistance and capacitance – measurement of high resistance by leakage – growth and decay of charge in a LCR circuit – condition for the discharge to be oscillatory – frequency of oscillation.

1. Growth of a current in a circuit containing a resistance and an inductance
2. Decay of a current in a circuit containing a resistance and an inductance
3. Growth of a charge in a circuit containing resistance and capacitance
4. Decay of a charge in a circuit containing resistance and a capacitance
5. Measurement of high resistance by leakage
6. Growth of charge in a LCR circuit – condition for the discharge to be oscillatory and frequency of oscillation.
7. Decay of charge in a LCR circuit – condition for the discharge to be oscillatory and frequency of oscillation.

12

Transient Currents

12.1. Growth of current in a circuit containing a resistance and inductance

Consider a circuit having an inductance L and a resistance R connected in series to a cell of steady emf E (Fig. 12.1). When the key K is pressed, there is a gradual growth of current in the circuit from zero to maximum value I_0. Let I be the instantaneous current at any instant.

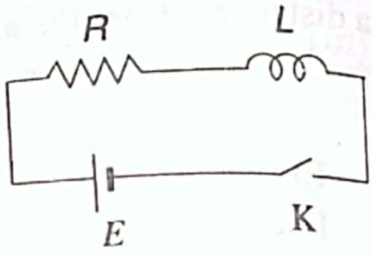


Fig. 12.1

Then, the induced back emf $\varepsilon = -L\frac{dI}{dt}$

$\therefore$

$$E = RI + L\frac{dI}{dt} \quad \text{... (1)}$$

When the current reaches the maximum value I_0, the back emf,

$$L\frac{dI}{dt} = 0.$$

Hence

$$E = RI_0 \quad \text{... (2)}$$

Substituting this value for E in Eq. (1),

$$RI_0 = RI + L\frac{dI}{dt} \quad \text{or} \quad R(I_0 - I) = L\frac{dI}{dt}$$

or

$$\frac{dI}{I_0 - I} = \frac{R}{L}dt$$

Integrating, $$-\log(I_0 - I) = \frac{R}{L}t + C \quad \text{... (3)}$$

where C is the constant of integration.

When $t = 0$, $I = 0$, $\therefore -\log_e I_0 = C$

Substituting this value of C in Eq. (3),

$$-\log(I_0 - I) = \frac{R}{L}t - \log_e I_0 \quad \text{or} \quad \log_e(I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

$$\log_e \frac{(I_0 - I)}{I_0} = -\frac{R}{L}t$$

$$\frac{I_0 - I}{I_0} = e^{-(R/L)t} \quad \text{or} \quad 1 - \frac{I}{I_0} = e^{-(R/L)t}$$

$$\therefore \qquad I = I_0\left(1 - e^{-(R/L)t}\right) \qquad \text{... (4)}$$

Eq. (4) gives the value of the instantaneous current in the *LR* circuit. The quantity (*L/R*) is called the *time constant* of the circuit.

$$\text{If } \frac{L}{R} = t, \quad I = I_0(1 - e^{-1}) = I_0\left(1 - \frac{1}{e}\right) = 0.632 I_0$$

Thus, *the time constant L/R of a L–R circuit is the time taken by the current to grow from zero to 0.632 times the steady maximum value of current in the circuit.*

Similarly, when $t = 2L/R$, $3L/R$ ..., the value of current will be 0.8647, 0.9502,... of the final maximum current.

When $t = 0$, $I = 0$ and when $t \to \infty$, $I = I_0$

Greater the value of *L/R*, longer is the time taken by the current *I* to reach its maximum value (Fig. 12.2).

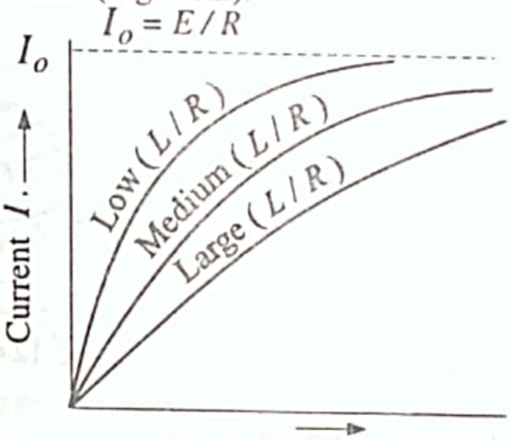


Fig. 12.2

12.2. Decay of Current in a Circuit Containing L and R

When the circuit is broken, an induced emf, equal to $-L\frac{dI}{dt}$ is again produced in the inductance *L* and it slows down the rate of decay of the current. The current in the circuit decays from maximum value I_0 to zero. During the decay, let *I* be the current at time *t*. In this case $E = 0$. The emf equation for the decay of current is

$$0 = RI + L\frac{dI}{dt} \qquad \text{... (1)}$$

$$\therefore \qquad \frac{dI}{I} = -\frac{R}{L}dt$$

Integrating, $\log_e I = -\frac{R}{L}t + C$ where C is a constant.

When $t = 0$, $I = I_0$, $\therefore \log_e I_0 = C$

$$\therefore \quad \log_e I = -\frac{R}{L}t + \log_e I_0, \text{ or } \log_e \frac{I}{I_0} = -\frac{R}{L}t$$

$$\frac{I}{I_0} = e^{-(R/L)t}$$

$$\therefore \quad I = I_0 e^{-(R/L)t} \qquad \dots (2)$$

Eq. (2) represents the current at any instant t during decay. A graph between current and time is shown in Fig. 12.3.

When $t = L/R$, $I = I_0 e^{-1} = \frac{1}{e} I_0 = 0.365\, I_0$

$t = 2L/R$, $I = I_0 e^{-2} = 0.1035\, I_0$

$t = 3L/R$, $I = I_0 e^{-3} = 0.05\, I_0$

Therefore, *the time constant L/R of a R–L circuit may also be defined as the time in which the current in the circuit falls to 1/e of its maximum value* when external source of e.m.f. is removed.

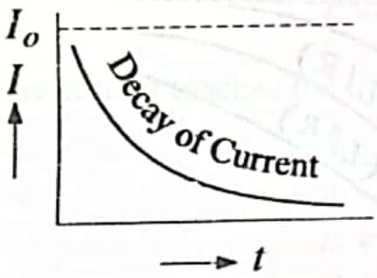


Fig. 12.3

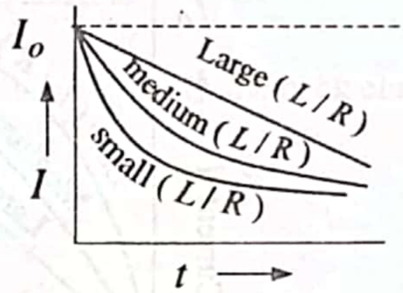


Fig. 12.4

The rate of decay of current is

$$\frac{dI}{dt} = -\frac{R}{L} I_0 e^{-(R/L)t} = -\frac{R}{L} I$$

Thus it is clear that greater the ratio *R/L, or smaller the time constant L/R, the more rapidly does the current die away* (Fig. 12.4).

Fig. 12.5 shows that the growth and decay curves are complementary.

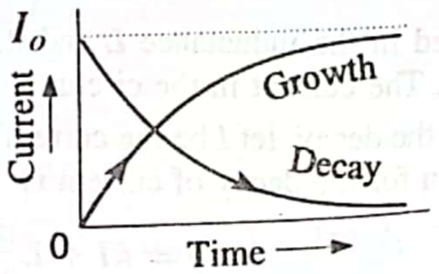


Fig. 12.5

Example 1. *An e.m.f. 10 volts is applied to a circuit having a resistance of 10 ohms and an inductance of 0.5 henry. Find the time required by the current to attain 63.2% of its final value. What is the time constant of the circuit ?*

Sol. $I = I_0 (1 - e^{-Rt/L})$

Given $\frac{I}{I_0} = \frac{63.2}{100}; \frac{R}{L} = \frac{10}{0.5} = 20$

$$\frac{63.2}{100} = 1 - e^{-20t}$$

$$e^{-20t} = 1 - 0.632 = 0.368$$

$$e^{20t} = \frac{1}{0.368} = 2.717$$

$$20t = \log_e 2.717$$

$$t = \frac{1}{20} \times 2.3026 \times \log_{10} 2.717$$

$$= \frac{2.3026 \times 0.4341}{20} = \mathbf{0.05\ sec.}$$

The time constant of the circuit is

$$\frac{L}{R} = \frac{0.5}{10} = \frac{1}{20}\text{ sec.}$$

Example 2. *An inductance of 500 mH and a resistance of 5 ohms are connected in series with an e.m.f of 10 volts. Find the final current. If now the cell is removed and the two terminals are connected together, find the current after (i) 0.05 sec. and (ii) 0.2 sec.*

Sol. Final current $I_0 = \frac{E}{R} = \frac{10}{5} = 2A$

During discharge, $I = I_0 e^{-Rt/L}$

Now $\frac{R}{L} = \frac{5}{500 \times 10^{-3}} = 10$

(*i*) When $t = 0.05$ *sec.*, $I = 2e^{-10 \times 0.05} = 1.213A$

(*ii*) When $t = 0.2$ *sec.*, $I = 2e^{-10 \times 0.2} = 0.271A$

12.3. Charge and Discharge of a Capacitor through a Resistor

(*a*) **Growth of Charge.** A capacitor C and a resistance R are connected to a cell of emf E through a Morse key K (Fig. 12.6). When the key is pressed, a momentary current I flows through R. At any instant t, let Q be the charge on the capacitor of capacitance C.

P.D. across capacitor $= Q/C$

P.D. across resistor $= RI$

The emf equation of the circuit is

$$E = (Q/C) + RI \quad \text{... (1)}$$

$$E = (Q/C) + R\,(dQ/dt)$$

$$\because I = dQ/dt.$$

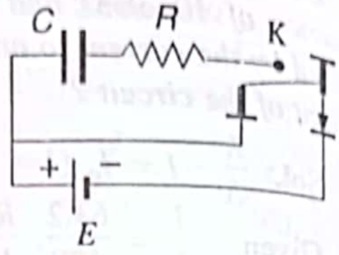


Fig. 12.6

The capacitor continues getting charged till it attains the maximum charge Q_0. At that instant $I = dQ/dt = 0$. The P.D. across the capacitor is $E = Q_0/C$.

i.e., when,
$$Q = Q_0, \frac{dQ}{dt} = 0 \text{ and } E = \frac{Q_0}{C}$$

$$\therefore \quad \frac{Q_0}{C} = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$(Q_0 - Q) = CR\frac{dQ}{dt}$$

$$\left(\frac{dQ}{Q_0 - Q}\right) = \frac{dt}{CR} \quad \text{... (2)}$$

Integrating, $-\log_e (Q_0 - Q) = \dfrac{t}{CR} + K$

where K is a constant.

When $t = 0$, $Q = 0 \quad \therefore -\log_e Q_0 = K$

$$\therefore \quad -\log_e (Q_0 - Q) = \frac{t}{CR} - \log_e Q_0$$

$$\log_e (Q_0 - Q) = -\frac{t}{CR} + \log_e Q_0$$

$$\log_e (Q_0 - Q) - \log_e Q_0 = -\frac{t}{CR}$$

$$\log_e\left(\frac{Q_0 - Q}{Q_0}\right) = -\frac{t}{CR}$$

$$\frac{Q_0 - Q}{Q_0} = e^{-\frac{t}{CR}} \quad \text{or} \quad 1 - \frac{Q}{Q_0} = e^{-\frac{t}{CR}}$$

$$\therefore \quad Q = Q_0\left(1 - e^{-\frac{t}{CR}}\right) \quad \text{... (3)}$$

The term CR is called time constant of the circuit.

At the end of time $t = CR$, $Q = Q_0\,(1 - e^{-1}) = 0.632Q_0$

Thus, the time constant may be defined as the time taken by the capacitor to get charged to 0.632 times its maximum value.

The growth of charge is shown in Fig. 12.7.

The rate of growth of charge is

$$\frac{dQ}{dt} = \frac{Q_0}{CR} e^{-\frac{t}{CR}} = \frac{1}{CR}(Q_0 - Q)$$

Thus it is seen that smaller the product CR, the more rapidly does the charge grow on the capacitor.

The rate of growth of the charge is rapid in the beginning and it becomes less and less as the charge approaches nearer and nearer the steady value.

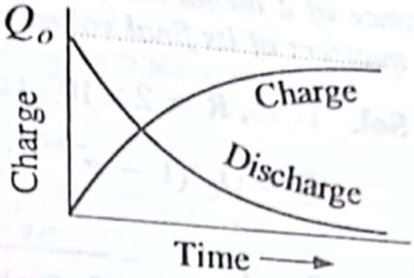


Fig. 12.7

(*b*) **Decay of charge (Discharging of a Capacitor through Resistance)**

Let the capacitor having charge Q_0 be now discharged by releasing the Morse key K (Fig. 12.6). The charge flows out of the capacitor and this constitutes a current. In this case $E = 0$.

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \qquad \text{... (1)}$$

or $$\frac{dQ}{Q} = -\frac{1}{CR}dt$$

Integrating, $$\log_e Q = -\frac{t}{CR} + K, \text{ where } K \text{ is a constant}$$

When $t = 0$, $Q = Q_0$ $\therefore$ $\log_e Q_0 = K$

$$\log_e Q = -\frac{t}{CR} + \log_e Q_0$$

or $$\log_e \frac{Q}{Q_0} = -\frac{t}{CR} \quad \text{or} \quad \frac{Q}{Q_0} = e^{-t/CR}$$

$\therefore$ $$Q = Q_0 e^{-t/CR} \qquad \text{... (2)}$$

This shows that the charge in the capacitor decays exponentially and becomes zero after infinite interval of time (Fig. 12.7).

The rate of discharge is

$$I = \frac{dQ}{dt} = -\frac{Q_0}{CR}e^{-t/CR} = -\frac{Q}{CR} \qquad \text{... (3)}$$

Thus, smaller the time-constant CR, the quicker is the discharge of the capacitor.

In Eq. (2), if we put $t = CR$, then $Q = Q_0 e^{-1} = 0.368\, Q_0$.

Hence *time constant may also be defined as the time taken by the current to fall from maximum to 0.368 of its maximum value.*

Example 1. *A capacitor is charged by DC supply through a resistance of 2 megaohms. If it takes 0.5 seconds for the charge to reach three quarters of its final value, what is the capacitance of the capacitor?*

Sol. Here, $R = 2 \times 10^6\ \Omega$, $t = 0.5\text{s}$, $Q/Q_0 = 3/4$.

$$Q = Q_0\,(1 - e^{-t/CR}) \quad \text{or} \quad \frac{Q}{Q_0} = (1 - e^{-t/CR})$$

$$\text{or} \quad \frac{3}{4} = \left(1 - e^{-\frac{0.5}{C \times (2 \times 10^6)}}\right)$$

$$\text{or} \quad e^{\frac{0.5}{C \times (2 \times 10^6)}} = 4 \quad \text{or} \quad \frac{0.5}{C \times (2 \times 10^6)} = \log_e 4$$

$$\text{or} \quad C = \frac{0.5}{(2.3026 \log_{10} 4)\ (2 \times 10^6)} = 0.18 \times 10^{-6}\ \text{F} = 0.18\ \mu\text{F}.$$

Example 2. *A capacitor of capacitance 0.1 μF is first charged and then discharged through a resistance of 10 megaohm. Find the time, the potential will take to fall to half its original value.*

Sol. $Q = Q_0\, e^{-t/CR}$ or $\dfrac{Q}{Q_0} = e^{-t/CR}$ or $\ln\left(\dfrac{Q_0}{Q}\right) = \dfrac{t}{CR}$

$$\therefore \quad t = CR \ln\left(\frac{Q_0}{Q}\right)$$

$$Q = CV \text{ and } Q_0 = CV_0. \qquad \therefore \quad \frac{Q_0}{Q} = \frac{V_0}{V}$$

$$\therefore \quad t = CR \ln\left(\frac{V_0}{V}\right)$$

Here, $C = 10^{-7}$ F; $R = 10^7\ \Omega$, $V_0/V = 2$.

$$\therefore \quad t = 10^{-7} \times 10^7 \times \ln 2 = 0.6931 \text{ s}.$$

Example 3. *A resistance R and a 2μF capacitor in series are connected to a 200 volt direct supply. Across the capacitor is a neon lamp that strikes at 120 volts. Calculate the value of R to make the lamp strike 5 seconds after switch has been closed.*

Sol. The resistance R must be such that the p.d. across the capacitor should rise to 120 volt in 5 seconds after the switch is closed. The lamp would then strike.

The equation of charging is

$$Q = Q_0\,(1 - e^{-t/RC})$$

or $$CV = CV_0 \left(1 - e^{-t/RC}\right)$$

or $$V = V_0 \left(1 - e^{-t/RC}\right)$$

Here, $V = 120$ volt, $V_0 = 200$ volt, $t = 5$ sec and

$$C = 2 \times 10^{-6} \text{ farad},$$

$$\therefore \quad 120 = 200 \left(1 - e^{-5/2 \times 10^{-6} R}\right)$$

or $$e^{5/2 \times 10^{-6} R} = \frac{5}{2}$$

$$\frac{5}{2 \times 10^{-6} R} = \log_e 5 - \log_e 2 = 1.6094 - 0.6931 = 0.9163$$

$$R = \frac{5}{2 \times 10^{-6} \times 0.9163} = 2.73 \times 10^{6} \text{ ohm} = \mathbf{2.73 \text{ megaohm}}$$

12.4. Measurement of High Resistance by Leakage

When a capacitor of capacitance C and initial charge Q_0 is allowed to discharge through a resistance R for a time t, the charge remaining on the capacitor is given by

$$Q = Q_0 e^{-t/CR}$$

$$\frac{Q_0}{Q} = e^{t/CR}$$

$$\log_e \frac{Q_0}{Q} = \frac{t}{CR}$$

$$\therefore \quad R = \frac{t}{C \log_e (Q_0/Q)} = \frac{t}{2.3026\, C \log_{10} (Q_0/Q)}$$

If R is high, CR will be high and the rate of discharge of capacitor will be very slow. Thus if we determine Q_0/Q from experiment, then R can be calculated.

Connections are made as shown in Fig. 12.8. C is a capacitor of known capacitance, R is the high resistance to be measured, B.G. is a ballistic galvanometer, E is a cell, and K_1, K_2, K_3 are tap keys.

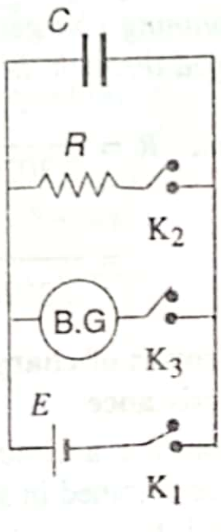


Fig. 12.8

Keeping K_2 and K_3 open, the capacitor is charged by depressing the key K_1. K_1 is then opened and at once K_3 is closed. The capacitor discharges through the galvanometer which records a throw θ_0. The throw θ_0 is proportional to Q_0. The capacitor is again charged to the maximum value keeping K_2 and K_3 open and closing K_1. K_1 is then opened and K_2 is closed for a *known* time *t*. Some of the charge leaks through *R*. K_2 is opened and at once K_3 is closed. The charge *Q* remaining on the capacitor then discharges through the galvanometer. The resulting throw θ is noted. Then $Q \propto \theta$

Now, $$\frac{Q_0}{Q} = \frac{\theta_0}{\theta}$$

$$\therefore \quad R = \frac{t}{2.3026\ C \log_{10} (\theta_0/\theta)}$$

A series of values of *t* and θ are obtained. A graph is plotted between *t* and $\log_{10} (\theta_0/\theta)$ which is a straight line. Its slope gives the mean value of $\frac{t}{\log_{10} (\theta_0/\theta)}$. As *C* is known, the value of *R* can be calculated.

Example 1. *If the charge on a capacitor of capacitance 2 μF in leaking through a high resistance of 100 megaohms is reduced to half its maximum value, calculate the time of leakage.*

Sol. $$R = \frac{t}{2.3026\ C \log_{10} (Q_0/Q)}$$

Here, $C = 2 \times 10^{-6}$ F, $R = 10^8\ \Omega$, $Q_0/Q = 2$.

$$\therefore \quad t = 2.3026\ CR \log_{10} (Q_0/Q) = 2.3026\ (2 \times 10^{-6})\ 10^8 \log_{10} 2$$
$$= 138.7 \text{ s.}$$

Example 2. *In an experiment to determine high resistance by leakage, a capacitor of 0.2 μF is used. It is first fully charged and discharged through a B.G. The observed kick was 12 cm on the scale. The capacitor was fully charged again and allowed to leak through R for 2 sec. The remaining charge in C gave a kick of 6 cm on the same scale when discharged through the B.G. Calculate R.*

Sol. $$R = \frac{t}{2.3026\ C \log_{10} (\theta_0/\theta)}$$

$$= \frac{2}{2.3026 \times (0.2 \times 10^{-6}) \log_{10} (12/6)} = 14.43 \text{ megaohms.}$$

12.5. Growth of charge in a Circuit with Inductance, Capacitance and Resistance

Consider a circuit containing an inductance *L*, capacitance *C* and resistance *R* joined in series to a cell of emf *E* (Fig. 12.9). When the key *K* is pressed, the capacitor is charged. Let *Q* be the charge on the capacitor and *I* the current in the circuit at an instant *t* during charging. Then, the p.d.

across the capacitor is Q/C and the self induced emf in the inductance coil is $L\ (dI/dt)$, both being opposite to the direction of E. The p.d. across the resistance R is RI.

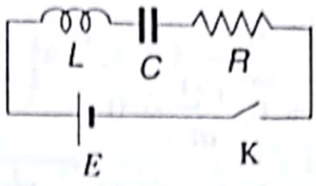


Fig. 12.9

The equation of emf's is

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = E \qquad \text{... (1)}$$

But $$I = \frac{dQ}{dt} \text{ and } \frac{dI}{dt} = \frac{d^2Q}{dt^2}$$

$$\therefore \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = E$$

or $$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q - CE}{LC} = 0$$

Putting $\frac{R}{L} = 2b$ and $\frac{1}{LC} = k^2$, we have

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + k^2 (Q - CE) = 0 \qquad \text{... (2)}$$

Let $x = Q - CE$. Then $\frac{dx}{dt} = \frac{dQ}{dt}$ and $\frac{d^2x}{dt^2} = \frac{d^2Q}{dt^2}$

Eq. (2) becomes, $$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + k^2x = 0 \qquad \text{... (3)}$$

Hence the most general solution of Eq. (3) is

$$x = Ae^{[-b + \sqrt{(b^2 - k^2)}]t} + Be^{[-b - \sqrt{(b^2 - k^2)}]t}$$

Now, $CE = Q_0$ = final steady charge on the capacitor.

$\therefore \quad x = Q - CE = Q - Q_0$.

Hence $$Q - Q_0 = Ae^{[-b + \sqrt{(b^2 - k^2)}]t} + Be^{[-b - \sqrt{(b^2 - k^2)}]t}$$

or $$Q = Q_0 + Ae^{[-b + \sqrt{(b^2 - k^2)}]t} + Be^{[-b - \sqrt{(b^2 - k^2)}]t} \qquad \text{... (4)}$$

Using initial conditions :

at $t = 0,\ Q = 0$ ... (5)

$\therefore \quad 0 = Q_0 + (A + B)$ or $A + B = -Q_0$

$$\frac{dQ}{dt} = A\left(-b + \sqrt{(b^2 - k^2)}\right] e^{[-b + \sqrt{(b^2 - k^2)}]t} +$$

$$B\left[-b - \sqrt{(b^2 - k^2)}\right] e^{[-b - \sqrt{(b^2 - k^2)}]t}$$

At $t = 0$, $\dfrac{dQ}{dt} = 0$

$$0 = A\left[-b + \sqrt{(b^2 - k^2)}\right] + B\left[-b - \sqrt{(b^2 - k^2)}\right]$$

$$\sqrt{(b^2 - k^2)}\,[A - B] = b(A + B) = -bQ_0$$

or $$A - B = -\frac{Q_0 b}{\sqrt{(b^2 - k^2)}} \qquad \text{... (6)}$$

Solving Eqs. (5) and (6),

$$A = -\frac{1}{2} Q_0 \left(1 + \frac{b}{\sqrt{(b^2 - k^2)}}\right) \qquad \text{... (7)}$$

$$B = -\frac{1}{2} Q_0 \left(1 - \frac{b}{\sqrt{(b^2 - k^2)}}\right) \qquad \text{... (8)}$$

Substituting the values of A and B in Eq. (4), we have

$$Q = Q_0 - \frac{1}{2} Q_0 e^{-bt} \left[\left(1 + \frac{b}{\sqrt{(b^2 - k^2)}}\right) e^{\sqrt{(b^2 - k^2)}\,.\,t} + \left(1 - \frac{b}{\sqrt{(b^2 - k^2)}}\right) e^{-\sqrt{(b^2 - k^2)}\,.\,t}\right] \qquad \text{... (9)}$$

Case I. If $b^2 > k^2$, $\sqrt{(b^2 - k^2)}$ is real. The charge on the capacitor grows exponentially with time and attains the maximum value Q_0 asymptotically, (curve 1 of Fig. 12.10). The charge is known as *over damped or dead beat.*

Case II. If $b^2 = k^2$, the charge rises to the maximum value Q_0 in a short time (curve 2 of Fig. 12.10). Such a charge is called *critically damped.*

Case III. If $b^2 < k^2$, $\sqrt{(b^2 - k^2)}$ is imaginary.

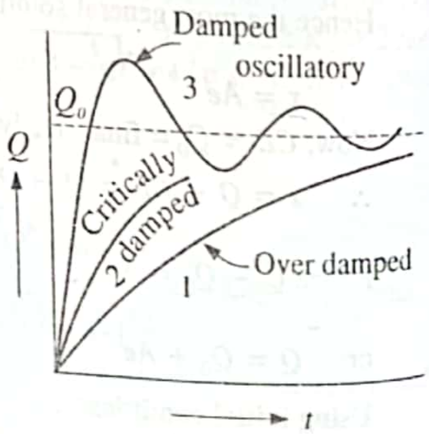


Fig. 12.10

Let $\sqrt{b^2 - k^2} = i\omega$ where $i = \sqrt{-1}$ and $\omega = \sqrt{k^2 - b^2}$

Eq. (9) may be written as

$$Q = Q_0 - \frac{1}{2} Q_0 e^{-bt} \left[\left(1 + \frac{b}{i\omega} \right) e^{i\omega t} + \left(1 - \frac{b}{i\omega} \right) e^{-i\omega t} \right]$$

$$Q = Q_0 - Q_0 e^{-bt} \left[\frac{e^{i\omega t} + e^{-i\omega t}}{2} + \frac{b}{\omega} \frac{(e^{i\omega t} - e^{-i\omega t})}{2i} \right]$$

$$Q = Q_0 - Q_0 e^{-bt} \left(\cos \omega t + \frac{b}{\omega} \sin \omega t \right)$$

$$Q = Q_0 \left[1 - \frac{e^{-bt}}{\omega} (\omega \cos \omega t + b \sin \omega t) \right]$$

Let $\omega = k \sin \alpha$ and $b = k \cos \alpha$ so that $\tan \alpha = \omega/b$.

$$Q = Q_0 \left[1 - \frac{e^{-bt}}{\omega} (k \sin \alpha \cos \omega t + k \cos \alpha \sin \omega t) \right]$$

or $$Q = Q_0 \left[1 - \frac{ke^{-bt}}{\omega} \sin (\omega t + \alpha) \right] \quad \text{... (10)}$$

$$Q = Q_0 \left[1 - \frac{e^{-\frac{R}{2L}t} \sqrt{\frac{1}{LC}}}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} . \sin \left[\left(\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \right) t + \alpha \right] \right]$$

This equation represents a damped oscillatory charge as shown by the curve (3). The charge oscillates above and below Q_0 till it finally settles down to Q_0 value. The frequency of oscillation in the circuit is given by

$$\nu = \frac{\omega}{2\pi} = \frac{\sqrt{k^2 - b^2}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

When $R = 0$, $\nu = \dfrac{1}{2\pi \sqrt{LC}}$

12.6. Discharge of a Capacitor through an inductor and a Resistor in series (Decay of charge in LCR circuit)

Consider a circuit containing a capacitor of capacitance C, an inductance L and a resistance R joined in series (Fig. 12.11). E is a cell K_2 is kept open. The capacitor is charged to maximum charge Q_0 by closing the key K_1. On opening K_1 and closing key K_2, the capacitor discharges through the inductance L and resistance R. Let I be the current in the circuit and Q be the charge in the capacitor at any instant during discharge. The circuit equation then is

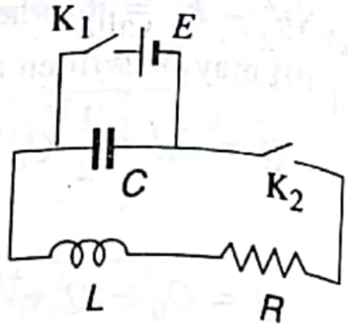


Fig. 12.11

$$L\frac{dI}{dt} + RI + \frac{Q}{C} = 0$$

But, $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$$\therefore \quad L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = 0 \qquad \text{... (1)}$$

Let $\frac{R}{L} = 2b$ and $\frac{1}{LC} = k^2$, then

$$\frac{d^2Q}{dt^2} + 2b\frac{dQ}{dt} + k^2Q = 0 \qquad \text{... (2)}$$

The general solution of this equation is

$$Q = A\,e^{(-b + \sqrt{b^2 - k^2})t} + B\,e^{(-b - \sqrt{b^2 - k^2})t} \qquad \text{... (3)}$$

where A and B are arbitrary constants.

When $t = 0$, $Q = Q_0$ and from Eq.(3), $A + B = Q_0$ ... (4)

$$\frac{dQ}{dt} = A(-b + \sqrt{b^2 - k^2})\,e^{(-b + \sqrt{b^2 - k^2})t}$$

$$+ B(-b - \sqrt{b^2 - k^2})\,e^{(-b - \sqrt{b^2 - k^2})t}$$

When, $t = 0$, $\frac{dQ}{dt} = 0$

$$\therefore A(-b + \sqrt{b^2 - k^2}) + B(-b - \sqrt{b^2 - k^2}) = 0$$

$$-b(A + B) + \sqrt{b^2 - k^2}\,(A - B) = 0$$

$$-bQ_0 + \sqrt{b^2 - k^2}\,(A - B) = 0$$

$$\therefore \quad A - B = \frac{bQ_0}{\sqrt{b^2 - k^2}} \qquad \text{... (5)}$$

From Eqs. (4) and (5), we get

$$A = \frac{1}{2}Q_0\left(1 + \frac{b}{\sqrt{(b^2 - k^2)}}\right) \text{ and } B = \frac{1}{2}Q_0\left(1 - \frac{b}{\sqrt{(b^2 - k^2)}}\right)$$

Putting these values of A and B in Eq. (3), we get

$$Q = \frac{1}{2} Q_0 e^{-bt} \left[\left(1 + \frac{b}{\sqrt{(b^2 - k^2)}} \right) e^{\sqrt{(b^2 - k^2)}\, t} + \left(1 - \frac{b}{\sqrt{(b^2 - k^2)}} \right) e^{-\sqrt{(b^2 - k^2)}\, t} \right] \quad \ldots (6)$$

Case I. If $b^2 > k^2$, $\sqrt{(b^2 - k^2)}$ is real and positive and the charge of the capacitor decays exponentially, becoming zero asymptotically (curve 1 of Fig. 12.12). This discharge is known as *over damped, non-oscillatory or dead beat.*

Case II. When $b^2 = k^2$, $Q = Q_0 (1 + bt) e^{-bt}$

This represents a *non-oscillatory* discharge.-This discharge is known as *criticall; damped* (Curve 2 of Fig. 12.12). The charge decreases to zero exponentially in a short time.

Case III. If $b^2 < k^2$, $\sqrt{(b^2 - k^2)}$ is imaginary.

$\sqrt{b^2 - k^2} = i\omega$, where $\omega = \sqrt{(k^2 - b^2)}$

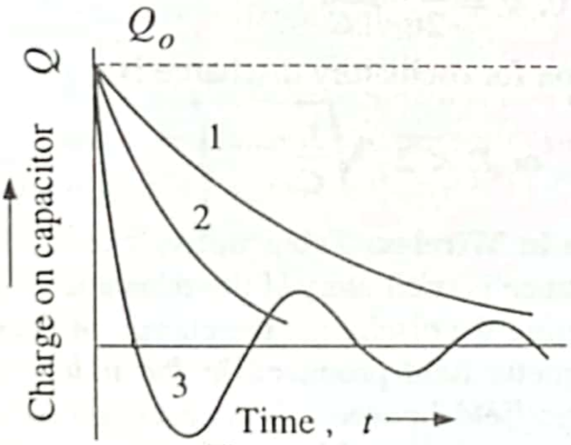


Fig. 12.12

$$\therefore \quad Q = \frac{1}{2} Q_0 e^{-bt} \left[\left(1 + \frac{b}{i\omega} \right) e^{i\omega t} + \left(1 - \frac{b}{i\omega} \right) e^{-i\omega t} \right]$$

$$= Q_0 e^{-bt} \left[\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right) + \frac{b}{\omega} \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i} \right) \right]$$

$$= \frac{Q_0 e^{-bt}}{\omega} (\omega \cos \omega t + b \sin \omega t)$$

Let $\omega = k \sin \alpha$ and $b = k \cos \alpha$ so that $\tan \alpha = \frac{\omega}{b}$.

$$Q = \frac{Q_0 e^{-bt} k}{\omega} (\cos \omega t \sin \alpha + \cos \alpha \sin \omega t)$$

$$= \frac{Q_0 e^{-bt} k}{\omega} \sin (\omega t + \alpha)$$

$$Q = \frac{Q_0 e^{-\frac{R}{2L}t}}{\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}\sqrt{LC}} \sin \left(\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}\, t + \alpha \right) \quad \text{... (7)}$$

This equation represents a *damped oscillatory* charge as shown by the curve (3). The charge oscillates above and below zero till it finally settles down to zero value.

The frequency of oscillation in the circuit is given by

$$\nu = \frac{\omega}{2\pi} = \frac{\sqrt{k^2 - b^2}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

When $R = 0$, $\nu = \frac{1}{2\pi \sqrt{LC}}$

The condition for oscillatory discharge is

$$\frac{R^2}{4L^2} < \frac{1}{LC}, \quad \text{or } R < 2\sqrt{\frac{L}{C}}$$

Importance in Wireless Telegraphy. The discharge of a capacitor through an inductance is oscillatory if the resistance R of the circuit is less than $2\sqrt{(L/C)}$. During the discharge, the energy of the charged capacitor is stored in the magnetic field produced in the inductance coil, then again back in the electric field between the capacitor plates, and so on. If the fields are caused to alternate rapidly, some energy escapes from the circuit permanently in the form of electro-magnetic waves which travel through space with the speed of light. These waves form the basis of wireless telegraphy.

Messages can be transmitted from one place to another with the help of codes.

Example 1. *If a battery, of emf 100 volts, is connected in series with an inductance of 10 mH, a capacitor of 0.05 μF and a resistance of 100 Ω, find (i) the frequency of the oscillatory current, (ii) the logarithmic decrement and (iii) the final capacitor charge.*

Sol. $\frac{R^2}{4L^2} = \frac{100^2}{4 \times (10^{-2})^2} = 2.5 \times 10^7$

and $\frac{1}{LC} = \frac{1}{10^{-2} \times 0.05 \times 10^{-6}} = 2 \times 10^9$

$R^2/4L^2 < 1/LC$. *Hence the charging of the capacitor is oscillatory.*

$$v = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)} = \frac{1}{2 \times 3.14}\sqrt{(2 \times 10^9 - 2.5 \times 10^7)}$$

$$= 7076.6 \text{ Hz.}$$

The logarithmic decrement $= \frac{RT}{2L} = \frac{100}{2 \times 10^{-2} \times 7076.6} = 0.707.$

The final capacitor charge $Q_0 = EC = 100 \times 0.05 \times 10^{-6} = 5\mu C.$

Example 2. *A charged capacitor of capacitance 0.01 μF is made to discharge through a circuit consisting of a coil of inductance 0.1 henry and an unknown resistance. What should be the maximum value of the unknown resistance, if the discharge of the capacitor is to be oscillatory.*

Sol. If R is the maximum value of the resistance for the discharge to be oscillatory, then

$$\frac{R^2}{4L^2} = \frac{1}{LC} \quad \text{or} \quad R = \sqrt{\frac{4L}{C}} = \sqrt{\left(\frac{4 \times 0.1}{0.01 \times 10^{-6}}\right)} = 6324\ \Omega$$

Example 3. *(i) Find out whether the discharge of a capacitor through a circuit containing the following elements, is oscillatory.*

C = 0.2 μF, L = 10 mH, R = 250 ohm

(ii) If so, find the frequency.

(iii) Calculate the maximum value of the resistance possible so as to make the discharge oscillatory.

Sol.

(*i*) The condition for oscillations is that

$$\frac{R^2}{4L^2} < \frac{1}{LC}$$

or $R^2 < \frac{4L}{C}$, or $R < \sqrt{\frac{4L}{C}}$

We have $\sqrt{\frac{4L}{C}} = \sqrt{\frac{4 \times 10 \times 10^{-3}}{0.2 \times 10^{-6}}} = 447$ ohm

It is given that $R = 250$ ohm

$\therefore R < \sqrt{\frac{4L}{C}}$

Therefore the discharge is oscillatory.

(*ii*) The frequency of oscillations is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{10\times10^{-3}\times0.2\times10^{-6}} - \frac{(250)^2}{4\times(10\times10^{-3})^2}}$$

$$= 54\times10^6 \text{ Hz.}$$

(3) The maximum possible resistance of oscillatory discharge is given by the equation

$$\frac{R^2}{4L^2} = \frac{1}{LC}$$

$$R = \sqrt{\frac{4L}{C}} = 447 \text{ ohm.}$$

EXERCISE XII

1. Derive Helmholtz equations for the growth and decay of current in a circuit having inductance and resistance.
2. Obtain an expression for the growth and decay of charge in a capacitor through a resistance.
3. Describe, with full theory, the method of measuring a high resistance by the leakage method.
4. A circuit is made up of a source of constant e.m.f., a self-inductance, an ohmic resistance, a capacitor and a key in series.

   Assuming the capacitor is uncharged before closing the key, investigate theoretically how its charge varies with time after closing the key.
5. A charged capacitor of capacitance C discharges through a circuit consisting of a coil of inductance L and a resistor R. Find the charge on the capacitor in t sec. after it starts discharging. Deduce the conditions under which the discharge is oscillatory. Find the period and frequency of the oscillatory discharge when R is very small.
6. A resistance R and an inductance L are connected to a battery of E volts. When will the potential difference across inductance be equal to that across resistance ?

   [**Hint.** Instantaneous pd across resistance $= RI = RI_0\,[1 - e^{-(R/L)t}]$

   Instantaneous pd across inductance $= L\dfrac{dI}{dt} = RI_0\, e^{-(R/L)t}$

   $\therefore\ 1 - e^{-(R/L)t} = e^{-(R/L)t}$

   or $t = \dfrac{L}{R}\log_e 2 = 0.69\,\dfrac{L}{R}$]
7. A two volt battery of negligibly small internal resistance is connected in series with a coil of inductance 1 henry and resistance 1 ohm. It

UNIT V

Alternating current – peak, average and RMS value of current and voltage – form factor – ac circuit containing resistance and inductance – choke coil – ac circuit containing resistance and capacitance – series and parallel resonance circuits –Q factor – power in an ac circuit containing LCR – Wattless current – Transformer – construction, theory and uses – energy loss – skin effect.

13

Alternating Current

13.1. EMF Induced in a Coil Rotating in a Magnetic Field

Consider a rectangular coil of N turns and of length a and width b rotating with uniform angular velocity ω about its axis in a uniform magnetic field **B** (Fig. 13.1).

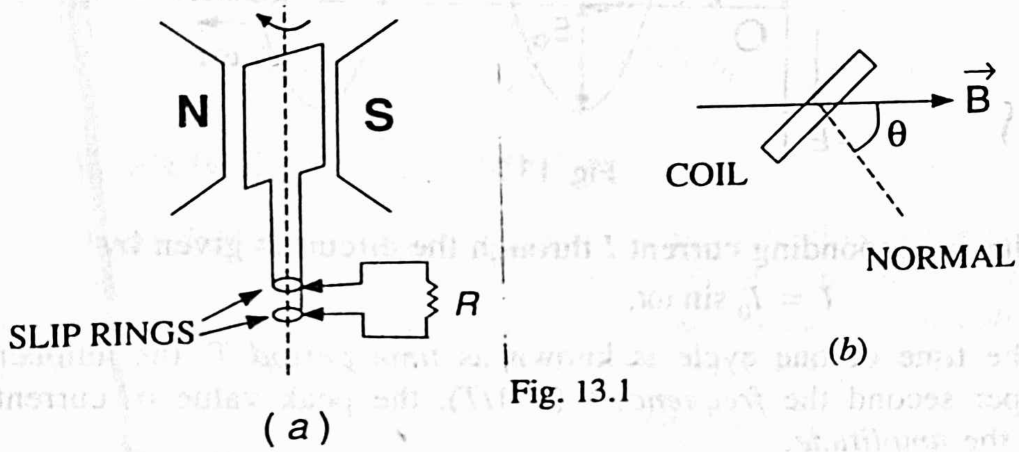


Fig. 13.1

The axis of rotation is at right angles to the field. As the coil rotates, the magnetic flux passing through it changes. Hence an emf is induced in the coil.

Suppose we start timing from the instant when the plane of the coil is at right angles to the field **B**, *i.e.*, when the angle between the normal to the plane of the coil and the direction of the field is zero. Then, at an instant t, the normal to the plane of the coil will make angle $\theta\ (= \omega t)$ with the direction of **B**.

The magnetic flux linked with N turns of the coil is

$$\phi = NBA \cos\theta = NBA \cos \omega t.$$

where $A\ (= ab)$ is the area of the coil.

The instantaneous induced emf,

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \omega t) = NBA\, \omega \sin \omega t = E_0 \sin \omega t$$

Here, $E_0 = NBA\omega$, called the *peak value* of the e.m.f.

Now, $\omega = 2\pi\nu$ where ν = frequency of alternating voltage.

Thus when $\omega t = 0$, $\sin \omega t = 0$ and $E = 0$

$\omega t = \pi/2$, $\sin \omega t = 1$ and $E = E_0$

$\omega t = \pi$, $\sin \omega t = 0$ and $E = 0$

$\omega t = 3\pi/2$, $\sin \omega t = -1$ and $E = -E_0$

and $\omega t = 2\pi$, $\sin \omega t = 0$ and $E = 0$ again.

A graph of E against ωt is a sine curve (Fig. 13.2). Such an e.m.f. is called an 'alternating e.m.f.' The resulting current in the coil, if the coil is part of a *closed circuit*, is the 'alternating current.'

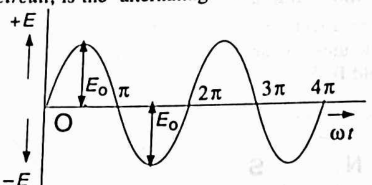


Fig. 13.2.

The corresponding current I through the circuit is given by

$$I = I_0 \sin \omega t.$$

The time of one cycle is known as *time period T*, the number of cycles per second the *frequency* ν (= $1/T$), the peak value of current or voltage the *amplitude*,

Peak Value of Alternating Current or emf. *The maximum value of alternating current or emf in the positive or negative direction is called peak value of alternating current or emf. It is denoted by I_0 or E_0.*

Mean Value of Alternating Current. *Mean value of alternating current is defined as its average over half a cycle.*

$$I_{mean} = \frac{\int_0^{T/2} I\,dt}{T/2} = \frac{\int_0^{\pi/\omega} I_0 \sin \omega t\,dt}{\pi/\omega}$$

$$= \frac{I_0\omega}{\pi}\left[\frac{-\cos \omega t}{\omega}\right]_0^{\pi/\omega}$$

$$= -\frac{I_0}{\pi}[\cos \pi - \cos 0]$$

$$= \frac{2I_0}{\pi} = 0.637\, I_0$$

Similarly, $E_{mean} = 0.637\, E_0$

Root mean square value of an alternating current. *It is defined as the square root of the average of I^2 during a complete cycle.*

$$\overline{I^2} = \frac{\int_0^{2\pi/\omega} I^2\,dt}{2\pi/\omega} = \frac{\int_0^{2\pi/\omega} I_0^2 \sin^2 \omega t\,dt}{2\pi/\omega}$$

$$= \frac{I_0^2\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1}{2}(1 - \cos 2\omega t)\,dt$$

$$= \frac{I_0^2\omega}{4\pi}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^{2\pi/\omega}$$

$$= \frac{I_0^2\omega}{4\pi}\left[\frac{2\pi}{\omega}\right] = \frac{I_0^2}{2}$$

$$I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Similarly, $E_{rms} = \dfrac{E_0}{\sqrt{2}} = 0.707\, E_0$

Form factor. The form factor gives an indication of the wave shape of the alternating voltage or current. *It is defined as the ratio of the virtual or r.m.s. value to the average value of alternating current or voltage.* Thus in the case of a sinusoidal current (or voltage), form factor is,

$$\text{Form factor} = \frac{I_{r.m.s.}}{I_{mean}} = \frac{E_{r.m.s.}}{E_{mean}} = \frac{0.707\, E_0}{0.637\, E_0} = 1.11$$

Effective value or virtual value of an A.C. *The rms value of an alternating current can also be defined as that direct current which produces the same rate of heating in a given resistance. Therefore, the r.m.s. value of alternating current is also called as the 'effective' or the 'virtual' value of the current.*

$$I_{virtual} = \frac{I_0}{\sqrt{2}} = I_{rms}$$

Suppose an alternating current of instantaneous value $I = I_0 \sin \omega t$ is flowing through a circuit of resistance R.

$$\left.\begin{array}{l}\text{Total quantity of heat produced}\\ \text{over the complete cycle}\end{array}\right\} = H = \int_0^T I^2 R\,dt \quad \text{... (1)}$$

Let I_v stand for the root mean square or virtual value of the current.

Then the heat produced in time T is given by

$$H = I_v^2 RT \quad \text{... (2)}$$

Comparing Eqs. (1) and (2),

$$I_v^2 RT = \int_0^T I^2 R\, dt \quad \text{or} \quad I_v^2 T = \int_0^T I^2\, dt$$

or
$$I_v^2 T = \int_0^T I_0^2 \sin^2 \omega t\, dt = \frac{I_0^2}{2}\int_0^T 2\sin^2 \omega t\, dt$$

$$= \frac{I_0^2}{2}\int_0^T (1 - \cos 2\omega t)\, dt = \frac{I_0^2}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 T}{2}$$

or
$$I_v^2 = \frac{I_0^2}{2}$$

or
$$I_v = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Similarly, *the r.m.s. value of an alternating voltage can be defined as that direct voltage which produces the same rate of heating. in a given resistance. The r.m.s. value of alternating voltage is also called as the 'effective' or the 'virtual' value of the voltage.*

$$E_{virtual} = \frac{E_0}{\sqrt{2}} = E_{rms}.$$

Impedance. *In any circuit the ratio of the effective voltage to the effective current is defined as the impedance Z of the circuit.*

13.2. AC Circuit Containing Resistance, Inductance and Capacitance in series (Series Resonance Circuit)

Let an alternating emf $E = E_0 \sin \omega t$ be applied to a circuit containing a resistance R, inductance L and capacitance C in series (Fig. 13.3). Let at any instant, I be the current in the circuit and Q be the charge on the capacitor.

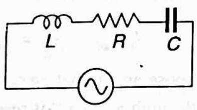


Fig. 13.3

The potential drop across the resistance = RI

The E.M.F. induced in the inductance = $L\frac{dI}{dt}$.

The potential across the plates of the capacitor = Q/C.

$$\therefore \quad L\frac{dI}{dt} + RI + \frac{Q}{C} = E_0 \sin \omega t.$$

Differentiating with respect to t,

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}\frac{dQ}{dt} = E_0\omega \cos \omega t. \quad \text{... (1)}$$

Let the trial solution be of the form

$$I = I_0 \sin(\omega t - \phi), \quad \text{... (2)}$$

where I_0 and ϕ are constants to be determined.

$$\frac{dI}{dt} = I_0\omega \cos(\omega t - \phi)$$

and
$$\frac{d^2I}{dt^2} = -I_0\omega^2 \sin(\omega t - \phi)$$

Substituting these values of I, $\frac{dI}{dt}$ and $\frac{d^2I}{dt^2}$, in Eq. (1), we get

$$-LI_0\omega^2 \sin(\omega t - \phi) + RI_0\omega \cos(\omega t - \phi) + \frac{I_0}{C}\sin(\omega t - \phi) = E_0\, \omega \cos \omega t$$

or
$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 \sin(\omega t - \phi) + R\omega I_0 \cos(\omega t - \phi)$$

$$= E_0\omega\, [\cos(\omega t - \phi) + \phi]$$

$$= E_0\omega\, [\cos(\omega t - \phi)\cos\phi - \sin(\omega t - \phi)\sin\phi]$$

Equating the coefficients of $\sin(\omega t - \phi)$ and $\cos(\omega t - \phi)$ on either side,

$$\left(-L\omega^2 + \frac{1}{C}\right) I_0 = -E_0\omega \sin\phi, \quad \text{... (3)}$$

and
$$R\omega I_0 = E_0\omega \cos\phi. \quad \text{... (4)}$$

Dividing Eq. (3) by Eq. (4), we get

$$\tan\phi = -\frac{\left(-L\omega^2 + \frac{1}{C}\right)}{R\omega} = \frac{\omega L - \frac{1}{C\omega}}{R} \quad \text{... (5)}$$

Squaring and adding Eqs. (3) and (4), we get

$$I_0^2\left[\left(-L\omega^2 + \frac{1}{C}\right)^2 + R^2\omega^2\right] = E_0^2\omega^2$$

or
$$I_0^2\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] = E_0^2$$

or $$I_0 = \frac{E_0}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}} \qquad \text{... (6)}$$

Substituting the value of I_0 in Eq. (2), we get

$$I = \frac{E_0}{\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}} \sin(\omega t - \phi), \qquad \text{... (7)}$$

where $\phi = \tan^{-1} \dfrac{\omega L - \frac{1}{\omega C}}{R}$.

Eq. (7) represents the current at any instant.

The quantity $\sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}$ is the *impedance Z* of the circuit.

$L\omega$ and $1/\omega C$ respectively represent inductive reactance X_L and capacitive reactance X_C. Thus $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$.

The current lags in phase behind e.m.f. by an angle

$$\phi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = \tan^{-1} \frac{X_L - X_C}{R}$$

The following three cases arise :

(*i*) When $X_L > X_C$, ϕ is positive so that the *current lags behind the applied emf.*

(*ii*) When $X_L < X_C$, ϕ is negative, so that the *current leads the applied emf.*

(*iii*) When $X_L = X_C$, $\phi = 0$, and the *current is in phase with the emf.*

Series Resonant Circuit

The value of current at any instant in a series *LCR* circuit is given by

$$I = \frac{E_0}{\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2\right\}}} \sin(\omega t - \phi) \qquad \text{... (1)}$$

where $\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{C\omega}\right)^2\right\}} = Z$

is called the impedance of the circuit.

At a particular frequency, $\omega L = \frac{1}{\omega C}$ so that the impedance becomes minimum being given by $Z = R$. *This particular frequency ν_0 at which the impedance of the circuit becomes minimum and, therefore the current becomes maximum, is called the resonant frequency of the circuit.* Such a circuit which admits maximum current is called *series resonant circuit.*

Thus at ν_0, we have

$$\omega L = \frac{1}{\omega C} \text{ or } 2\pi\nu_0 L = \frac{1}{2\pi\nu_0 C}$$

or $$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

The maximum current in the circuit $= I_0 = E_0/R$. The variation of current with frequency of applied voltage is shown in Fig. 13.4. The sharpness of peak depends upon the resistance R of the circuit. For low resistance, the peak is sharp.

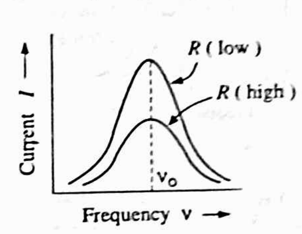


Fig. 13.4

Acceptor Circuit. The series resonant circuit is often called an 'acceptor' circuit. By offering *minimum impedance* to currents at the resonant frequency, it is able to select or accept most readily the current of this *one* frequency from among those of many frequencies.

In radio receivers, the resonant frequency of the circuit is tuned (by changing C) to the frequency of the signal desired to be detected.

Voltage Magnification. At resonance, the (peak) current through the circuit is

$$I_0 = \frac{E_0}{R}.$$

The (peak) voltage across the inductance is

$$V_L = X_L I_0 = \omega L \times \frac{E_0}{R} = \frac{\omega L}{R}(E_0) = Q E_0,$$

where Q is known as the *quality factor* of the circuit. Thus at resonance the voltage drop across L (which is also the voltage across C) is Q times the applied voltage. Hence the chief characteristic of the series resonant circuit is 'voltage magnification'. This voltage magnification does not increase the power in the circuit as the reactive component of the power is *wattless*.

The Q-factor

$$\text{Q-factor} = \frac{\text{Reactance of the coil at resonance}}{\text{Resistance of the circuit}} = \frac{L\omega_0}{R}$$

Q-factor determines the degree of selectivity of the circuit while tuning. This is because, for larger values of Q-factor the frequency response curve of the circuit is a steep narrow peak. For smaller values of Q-factor, the frequency response curve is quite flat (Fig. 13.5).

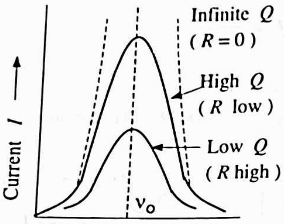


Fig. 13.5

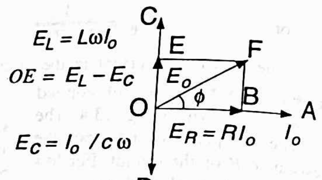


Fig. 13.6

Vector Diagram. A vector diagram of a series LCR circuit is shown in Fig. 13.6. Since $L - C - R$ are connected in series, the current through each is same. Let E_R, E_L and E_C be the potential drops across resistance, inductance and capacitance.

The vector $E_R = I_0 R$ is *in phase with* the current.

The vector $E_L = \omega L I_0$ is 90° *advance of* the current.

The vector $E_C = I_0/\omega C$, is 90° *behind* the current.

Let the vectors OB, OC and OD represent E_R, E_L and E_C. If $E_L > E_C$, the resultant of these two is $(E_L - E_C)$. This is represented by OE.

$$OE = OC - OD$$

$$= I_0\left(L\omega - \frac{1}{C\omega}\right);\ \text{where } L\omega > \frac{1}{C\omega}$$

$$E_0 = [OB^2 + BF^2]^{\frac{1}{2}} = [OB^2 + OE^2]^{\frac{1}{2}}$$

$$= \left[I_0^2R^2 + I_0^2\left(L\omega - \frac{1}{C\omega}\right)^2\right]^{\frac{1}{2}}$$

$$= I_0\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]^{\frac{1}{2}}$$

$$I_0 = \frac{E_0}{\sqrt{\left[\left(L\omega - \frac{1}{C\omega}\right)^2 + R^2\right]}}$$

The current lags behind the applied voltage by ϕ given as

$$\phi = \tan^{-1}\left(\frac{L\omega - 1/C\omega}{R}\right)$$

j operator method

Use of operator j in study of A.C. Circuits

The operator j is defined as a quantity which is numerically equal to $\sqrt{(-1)}$ and which represents the rotation of a vector through 90° in anti-clockwise direction. $-j$ represents the rotation of a vector through 90° in clockwise direction. The above facts about j are helpful in studying the A.C. circuits. We know that in A.C. circuits, E_L and E_C always lie at 90° in anticlockwise and clockwise direction respectively with respect to E_R (Fig. 13.7). Hence total emf of a circuit having L, C, R will be

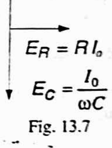


Fig. 13.7

$$E = E_R + jE_L - jE_C$$

Ordinarily a source of alternating e.m.f. E is denoted by $E_0 \sin \omega t$.

This is actually the imaginary part of the complex form of alternating e.m.f.,

$$E = E_0 e^{j\omega t}$$

The instantaneous current in the A.C. circuit has been expressed as $I = I_0 \sin(\omega t - \phi)$. This expression is the imaginary part of the complex current given by

$$I = I_0 e^{j(\omega t - \phi)}$$

Since the voltage across the inductor leads the current passing through it by 90°, the inductive reactance ωL can be written as $j\omega L$. $Z_L = j\omega L = jX_L$.

Since the voltage across the capacitor lags the current passing through it by 90°, the capacitive reactance $1/\omega C$ can be written as $-j/\omega C = 1/j\omega C$. $Z_c = \frac{1}{j\omega C} = -\frac{j}{\omega C} = -jX_C$.

A complex impedance can be written as the sum of a real term and imaginary term which are to be called resistance and complex reactance respectively,

$$Z = R + jX$$

where $X = X_L - X_C$ is the effective reactance in the circuit.

LCR Circuit (Series Resonance Circuit)

Consider a circuit containing an inductance L, a capacitance C and a resistance R joined in series. This series circuit is connected to an AC supply given by

$$E = E_0 e^{j\omega t} \quad \text{(Fig. 13.8)} \qquad \ldots (1)$$

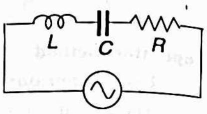


Fig. 13.8

The total complex impedance is

$$Z = Z_R + Z_L + Z_C$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= \sqrt{R^2 + (\omega L - 1/\omega C)^2}\, e^{j\phi} \qquad \ldots (2)$$

where $\tan\phi = \frac{(\omega L - 1/\omega C)}{R}$

Using Ohm's law in complex form, the 'complex' current in the circuit is

$$I = \frac{E}{Z} = \frac{E_0 e^{j\omega t}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\, e^{j\phi}}$$

$$\therefore \quad I = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\, e^{j(\omega t - \phi)} \qquad \ldots (3)$$

$$\text{But } I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \quad I = I_0 e^{j(\omega t - \phi)} \qquad \ldots (4)$$

The actual emf is the imaginary part of the equivalent complex emf. Hence the actual current in the circuit is obtained by taking the imaginary part of the above 'complex' current.

$$\therefore i = Im.(I) = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin(\omega t - \phi) \qquad \ldots (5)$$

The equivalent impedance of the series LCR circuit is

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The current 'lags' behind the voltage by an angle

$$\phi = \tan^{-1}\frac{(\omega L - 1/\omega C)}{R}$$

Example 1. *A circuit consists of a non-inductive resistance of 50 ohms, an inductance of 0.3 henry, and a resistance of 2 ohms and a capacitor of 40 micro-farad in series and is supplied with 200 volts at 50 Hz. Find the impedance, the current, lag or lead, and the power in the circuit.*

Sol. Total resistance $= R = 50 + 2 = 52\ \Omega$

$C = 40 \times 10^{-6}$ F, $L = 0.3$ H, $\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$,

$E_{r.m.s.} = 200$ V.

$$\text{Impedance} = Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$= \sqrt{(52)^2 + \left(0.3 \times 100\pi - \frac{1}{40 \times 10^{-6} \times 100\pi}\right)^2}$$

$$= 53.97\ \Omega$$

$\therefore$ Current $I_{rms} = \dfrac{E_{rms}}{Z} = \dfrac{200}{53.97} = 3.71\ A.$

$\therefore \quad I_{max} = 3.71 \times \sqrt{2}\ A$

Now, $\quad X_L = \omega L = (100\,\pi) \times 0.3 = 94.2\ \Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{(40 \times 10^{-6}) \times 100\,\pi} = 79.6\Omega$$

Since $X_L > X_C$ the current lags behind the applied e.m.f.

$$\phi = \tan^{-1} \frac{\omega L - 1/\omega C}{R} = \tan^{-1}\left(\frac{94.2 - 79.6}{52}\right) = \tan^{-1}\ (0.28)$$

$$= 15^\circ\ 39'$$

Power factor $\cos\phi = \dfrac{R}{\sqrt{[R^2 + (\omega L - 1/\omega C)^2]}}$

$$= \frac{52}{[(52)^2 + (14.6)^2]^{1/2}} = 0.964$$

Apparent power $= E_{rms} \times I_{rms} = 200 \times 3.71 = 742$ V.A

True power = Apparent power $\times$ Power factor $= 742 \times 0.964$

$= 716$ watt

Example 2. *An alternating potential of 100 volt and 50 hertz is applied across a series circuit having an inductance of 5 henry, a resistance of 100 ohm and a variable capacitance. At what value of capacitance will the current in the circuit be in phase with the applied voltage ? Calculate the current in this condition. What will be the potential differences across the resistance, inductance and capacitance ?*

Sol. For resonance, $\omega L = \dfrac{1}{\omega C}$ or $C = \dfrac{1}{\omega^2 L}$

Here, $\omega = 2\pi\nu = 100\,\pi$, $L = 5$ H

$$\therefore C = \frac{1}{(100\pi)^2 \times 5} = 2 \times 10^{-6}\ \text{farad} = 2\mu F$$

$$\left.\begin{array}{l}\text{The current}\\ \text{at resonance}\end{array}\right\} = I_{rms} = \frac{E_{rms}}{R} = \frac{100}{100} = 1.0A$$

P.D. across $R = I_{rms} \times R = 1.0 \times 100 =$ **100 volt (rms),**

P.D. across $L = I_{rms} \times \omega L = 1.0 \times (100\pi) \times 5 =$ **1570 volt (rms)**

P.D. across $C = I_{rms} \times \dfrac{1}{C\omega} = 1.0 \times \dfrac{1}{(2 \times 10^{-6}) \times 100\,\pi}$

$=$ **1570 volt (rms).**

The voltages across the inductor and capacitor are much greater than the applied voltage. But they differ in phase by 180°. So their algebraic sum is zero.

13.3. Parallel Resonant Circuit

Here, capacitor C is connected in parallel to the series combination of resistance R and inductance L. The combination is connected across the AC source (Fig. 13.9). The applied voltage is sinusoidal, represented by

$$E = E_0\, e^{j\omega t}$$

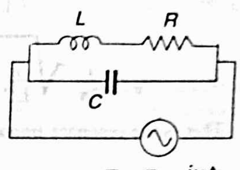


Fig. 13.9

Complex impedance of L - branch

$$Z_1 = R + jL\omega$$

Complex impedance of C - branch

$$Z_2 = \frac{1}{jC\omega}$$

Z_1 and Z_2 are in parallel.

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + \frac{1}{1/j\omega C} = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{(R + j\omega L) \times (R - j\omega L)} + j\omega C$$

$$= \frac{R}{R^2 + (L\omega)^2} + j\left[C\omega - \frac{L\omega}{R^2 + (L\omega)^2}\right]$$

The current $I = E/Z = E \times \dfrac{1}{Z}$

$$\therefore \quad I = E\left[\frac{R}{R^2 + (L\omega)^2} + j\left(C\omega - \frac{L\omega}{R^2 + (L\omega)^2}\right)\right]$$

Let $A\cos\phi = \dfrac{R}{R^2 + (L\,\omega)^2}$; $A\sin\phi = C\omega - \dfrac{L\omega}{R^2 + (L\omega)^2}$

$$\therefore \quad I = E\,(A\cos\phi + jA\sin\phi) = EAe^{j\phi} = E_0 A\, e^{j(\omega t + \phi)}$$

$$\text{where} \quad \phi = \tan^{-1}\frac{C\omega - \left(\dfrac{L\omega}{R^2 + (L\omega)^2}\right)}{\left(\dfrac{R}{R^2 + (L\omega)^2}\right)};$$

$$A^2 = \frac{R^2}{(R^2 + \omega^2 L^2)^2} + \left(C\omega - \frac{L\omega}{R^2 + \omega^2 L^2}\right)^2$$

The magnitude of the admittance

13.4. Power in ac circuit containing resistance, inductance and capacitance

Consider an ac circuit containing resistance, inductance and capacitance. E and I vary continuously with time. Therefore power is calculated at any instant and then its mean is calculated over a complete cycle.

The instantaneous values of the voltage and current are given by

$$E = E_0 \sin \omega t,$$

$$I = I_0 \sin (\omega t - \phi).$$

where ϕ is the phase difference between current and voltage.

Hence power at any instant is

$$E \times I = E_0 I_0 \sin \omega t \sin (\omega t - \phi)$$

$$= \frac{1}{2} E_0 I_0 [\cos \phi - \cos (2\omega t - \phi)]. \quad \text{... (1)}$$

Average power consumed over one complete cycle is

$$P = \frac{\int_0^T E\,I\,dt}{\int_0^T dt}$$

$$= \frac{\int_0^T \frac{1}{2} E_0 I_0 [\cos \phi - \cos (2\omega t - \phi)]\, dt}{T}$$

$$= \frac{1}{2} \frac{E_0 I_0}{T} \left[(\cos \phi)\, t - \frac{\sin (2\omega t - \phi)}{2\omega} \right]_0^T$$

$$= \frac{1}{2} \frac{E_0 I_0}{T} \left[(\cos \phi)\, T - 0 - \frac{\sin (2\omega T - \phi)}{2\omega} + \frac{\sin (-\phi)}{2\omega} \right]$$

Now $T = \frac{2\pi}{\omega}$ and $\sin (4\pi - \phi) = \sin (-\phi)$

$$P = \frac{1}{2} \frac{E_0 I_0 \omega}{2\pi} \left[(\cos \phi) \frac{2\pi}{\omega} - \frac{\sin (-\phi)}{2\omega} + \frac{\sin (-\phi)}{2\omega} \right]$$

$$= \frac{1}{2} E_0 I_0 \cos \phi$$

$$= \frac{E_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} \times \cos \phi$$

$$= E_{r.m.s.}\, I_{r.m.s.} \cdot \cos \phi \quad \text{... (2)}$$

average power = (virtual volts) × (virtual amperes) × cos ϕ

The term (*Virtual volts × Virtual amperes*) is called *apparent power* and cos ϕ is called the *power factor*. Thus

True power = apparent power × power factor

Evidently, the *power factor is the ratio of the true power to the apparent power.*

As cos ϕ is the factor by which the product of the r.m.s. values of the voltage and current must be multiplied to give the power dissipated, it is known as the 'power factor' of the circuit. For a circuit containing resistance, capacitance and inductance in series,

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

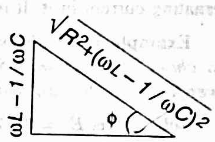


Fig. 13.11

From Fig. 13.11, the expression for the power factor is

$$\cos \phi = \frac{R}{\sqrt{\left\{ R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right\}}}$$

Special cases. (1) In a *purely resistive circuit*, $\phi = 0$ or $\cos \phi = 1$.

∴ true power = $E_v \times I_v$.

(2) In a *purely inductive circuit*, current lags behind the applied emf by 90° so that $\phi = 90°$ or $\cos \phi = 0$.

Thus true power consumed = 0

(3) In a *purely capacitive circuit*, current leads the applied voltage by 90° so that $\phi = -90°$ or $\cos (-90°) = \cos 90° = 0$

∴ true power = 0

(4) In an ac circuit containing a resistance and inductance in series,

$$\textit{Power factor}, \cos \phi = \frac{R}{\sqrt{R^2 + (L\omega)^2}}$$

(5) In an ac circuit containing a capacitance C and a resistance R in series,

$$\cos \phi = \frac{R}{\sqrt{\left(\frac{1}{C^2 \omega^2} + R^2 \right)}}$$

13.5. Wattless current

The average power dissipated during a complete cycle is $E_v I_v \cdot \cos\phi$

The current in A.C. circuit is said to be wattless when the average power consumed in the circuit is zero.

If an ac circuit is purely inductive or purely capacitive with no ohmic resistance, phase angle $\phi = \pi/2$ so that $\cos\phi = 0$ or the power consumed is zero. The current in such a circuit does not perform any useful work and is rightly called the *wattless* or *idle current*. In this situation, the circuit does not consume any power, though it offers a resistance to the flow of alternating current in it. It is the principle of choke coil.

Example 1. *An alternating voltage of 10 volts at 100 Hz is applied to a choke of inductance 5 henry and of resistance 200 ohms. Find the power factor of the coil and the power absorbed.*

Sol. Here, $E_v = 10\,V$, $\nu = 100\,Hz$, $L = 5\,H$, and $R = 200\,\Omega$

$$\text{Power factor, } \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{200}{\sqrt{(200)^2 + (2\pi \times 100 \times 5)^2}}$$

$$= 0.062$$

$$\text{Power absorbed} = E_v \cdot I_v \cdot \cos\phi = E_v \cdot \frac{E_v}{Z} \cos\phi$$

$$= \frac{(E_v)^2}{\sqrt{R^2 + \omega^2 L^2}} \cos\phi = \frac{10 \times 10\,(0.062)}{\sqrt{(200)^2 + (2\pi \times 100 \times 5)^2}}$$

$$= 0.00189\text{ W}$$

Example 2. *A source of e.m.f. 50 V, r.m.s. is connected across an air cored coil. When the supply frequency is 50 Hz the power consumed is found to be 100 W, whereas when the frequency is increased to 100 Hz, the power becomes 50 W. Find the self-inductance and the resistance of the coil.*

Sol. Let the inductance and resistance required be L and R respectively.

$$I_v = \frac{E_v}{\sqrt{(R^2 + \omega^2 L^2)}}$$

When the supply frequency is 50 Hz, $\omega = 2\pi \times 50. = 100\pi$

$$I_v' = \frac{50}{\sqrt{R^2 + (100\pi)^2 L^2}}$$

The power consumed is dissipated only in the circuit resistance. It is,

$$I_v'^2 R = \frac{50^2}{[R^2 + (100\pi)^2 L^2]} R = 100W \qquad \text{... (1)}$$

Let I_v'' be the current when the frequency is 100 Hz.

The power consumed now is

$$I_v''^2 R = \frac{50^2 R}{[R^2 + (200\pi)^2 L^2]} = 50W \qquad \text{... (2)}$$

From Eq. (1), $2500\,R = 100\,[R^2 + 10^4 \pi^2 L^2]$ ... (3)

From Eq. (2), $2500\,R = 50\,[R^2 + 4 \times 10^4 \pi^2 L^2]$ ... (4)

From (3) and (4), $R^2 = 2 \times 10^4 \times \pi^2 L^2$

or $R = 100\sqrt{2}\,\pi L$ ... (5)

Substituting for L^2 in Eq. (3),

$$2500\,R = 100\left[R^2 + \frac{10^4 \pi^2 R^2}{2 \times 10^4 \times \pi^2}\right]$$

$$\therefore \quad R = 16.7\,\Omega$$

$$\text{From Eq. (5), } L = \frac{R}{100\sqrt{2}\,\pi} = \frac{16.7}{100\sqrt{2}\,\pi} = 0.0373\ H = 37.3\ mH$$

Example 3. *What do you understand by wattless and power components of an alternating current ?*

A coil of self-inductance 0.7 H is joined in series with a non-inductive resistance of 50 Ω. Calculate the wattless and power components as well as the total current when connected to a supply of 200 volt at a frequency of 50 Hz.

Sol. In a circuit containing an inductance L and a resistance R in series, the current lags behind the emf by an angle ϕ, where $\tan\phi = \omega L/R$.

Let the vector E represent the rms value of emf and the vector I the rms value of current (Fig. 13.12)

Component of I parallel to $E = I\cos\phi$

Component of I perpendicular to $E = I\sin\phi$

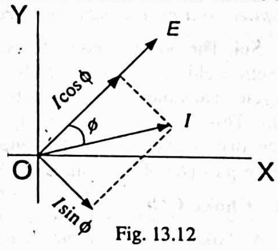


Fig. 13.12

The component $I\cos\phi$, which is in phase with the emf is responsible for the power dissipated in the circuit and is called the *power* component of the current. The component $I\sin\phi$, which lags 90° in phase behind the emf does not contribute to the power and is called the *idle* or *wattless* component of the current.

Here, $L = 0.7$ H, $R = 50\ \Omega$, $E_{rms} = 200$ V, $\omega = 2\pi\nu = 2\pi \times 50$

$$\text{Impedance, } Z = \sqrt{(R^2 + \omega^2 L^2)} = \sqrt{[(50)^2 + (2\pi \times 50)^2\,(0.7)^2]}$$

$$= 225.6\ \Omega$$

$\therefore$ current, $I_{rms} = \dfrac{E_{rms}}{Z} = \dfrac{200}{225.6} = 0.89$ A

Power component of current is

$$I_{rms} \cos\phi = I_{rms} \times \frac{R}{\sqrt{(R^2 + \omega^2 L^2)}} = 0.89 \times \frac{50}{225.6}$$

$$= 0.20 \text{ A}$$

The wattless component of the current is

$$I_{rms} \sin\phi = I_{rms} \times \frac{\omega L}{\sqrt{(R^2 + \omega^2 L^2)}} = 0.89 \times \frac{220}{225.6}$$

$$= 0.87$$

Example 4. *A coil has an inductance of 0.1 H and a resistance of 12 ohms. It is connected to a 220 V, 50 Hz mains. Determine the (1) reactance of the coil, (2) impedance of the coil, and (3) the reading of a wattmeter.*

Sol. Reactance of coil = $L\omega = 0.1 \times 2\pi \times 50\,\Omega = 31.43\,\Omega$

Z = Impedance of coil = $\sqrt{R^2 + L^2\omega^2}$

$$= \sqrt{(12)^2 + 4\pi^2 \times (0.1)^2 \times (50)^2} = 33.6\,\Omega$$

Reading of watt meter $= \bar{P} = \dfrac{E_{rms}^2}{Z} \cos\phi$

$$= \frac{(220)^2}{33.6} \times \left(\frac{12}{33.6}\right)$$

$$= 514.5 \text{ W}$$

Example 5. *There is no dissipation of power when an alternating emf is applied to a purely inductive circuit. Explain.*

Sol. The power used by the source in one part of a cycle to create the magnetic field of L is delivered back to the source in the remaining part of the cycle. Thus the inductance acts like a storage of magnetic energy in the circuit. This storage gets filled up and emptied alternately, as the voltage source drives the circuit. No energy is wasted in this process. In the resistive part (R) of the coil, some losses occur by way of heat dissipation.

13.6. Choke Coil

A choke coil is an inductance coil which is used to control the current in an ac circuit.

Construction. A choke consists of a coil of several turns of insulated thick copper wire of low resistance but large inductance, wound over a laminated core (Fig. 13.13). The core is layered and is made up of thin sheets of stalloy to reduce hysteresis losses. The laminations are coated with shellac to insulate and bound together firmly so as to minimise loss of

energy due to eddy currents.

Principle.

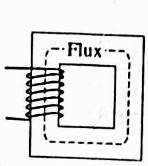


Fig. 13.13

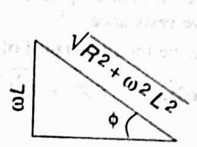


Fig. 13.14

The average power dissipated in the choke coil is given by

$$P = \frac{1}{2} E_0 I_0 \cos\phi$$

If the resistance of the choke coil is R and the inductance of the choke coil is L, then the power factor $\cos\phi$ is given by

$$\cos\phi = \frac{R}{\sqrt{(R^2 + \omega^2 L^2)}} \qquad \text{(Fig. 13.14)}$$

The inductance L of the choke coil is quite large on account of its large number of turns and the high permeability of iron core, while its resistance R is very small. Hence $\cos\phi$ is nearly zero. Therefore, the power absorbed by the coil is extremely small. Thus *the choke coil reduces the strength of the current without appreciable wastage of energy.* The only waste of energy is due to the hysteresis loss in the iron core. The loss due to eddy currents is minimised by making the core *laminated.*

Preference of choke coil over an ohmic resistance for diminishing the current. The current in an A.C. circuit can also be diminished by using an ordinary ohmic resistance (rheostat) in the circuit. But such a method of controlling A.C. is not economical as much of the electrical energy (I^2Rt) supplied by the source is wasted as heat. Hence the choke coil is to be preferred over the ohmic resistance.

The energy used in establishing the magnetic field in the choke coil is restored when the magnetic field collapses. Hence to regulate ac, it is more economical to use a choke than a resistance.

Choking coils are very much used in electronic circuits, mercury lamps and sodium vapour lamps.

Example 1. *An electric lamp which runs at 100 volts D.C. and 10 amp. current is connected to 220 volts 50 Hz A.C. mains. Calculate the inductance of the choke in the circuit.*

Sol. Resistance of the lamp $R = \frac{V}{I} = \frac{100}{10} = 10\,\Omega$.

If the lamp is to be run from 220 volts, 50 Hz A.C. mains a choke (inductance) should be placed in series with the lamp in order to increase its effective resistance.

Let L be the inductance of the required choke. Then

$$\text{Impedance} = \sqrt{(R^2 + \omega^2L^2)} = \sqrt{[(10)^2 + (2\pi \times 50)^2L^2]}$$
$$= \sqrt{(100 + 10^4\pi^2L^2)}.$$

Now
$$\text{current} = \frac{\text{voltage}}{\text{impedance}}$$

$\therefore$
$$10 = \frac{220}{\sqrt{(100 + 10^4\pi^2L^2)}}$$

$\therefore$
$$L = 0.062\ H.$$

Example 2. *A 20 V, 5 W lamp is to be used on ac mains of 200 V, 50 Hz. Calculate the (i) capacitor, (ii) inductor, to be put in series to run the lamp. How much pure resistance should be included in place of the above devices so that the lamp can run on its rated voltage ? Which of the above arrangements will be more economical and why ?*

Sol. Current required by the lamp $= I = \frac{\text{wattage}}{\text{voltage}} = \frac{5}{20} = 0.25\,A$

Resistance of the lamp $= R = \frac{\text{voltage}}{\text{current}} = \frac{20}{0.25} = 80\,\Omega$

(*i*) When a capacitor of value C farads is put in series with the lamp,

$$\text{the impedance of the circuit} = \sqrt{\left(R^2 + \frac{1}{\omega^2C^2}\right)}$$

$$I = \frac{E}{\sqrt{(R^2 + 1/\omega^2C^2)}}$$

or
$$\frac{200}{\sqrt{\left(80^2 + \frac{1}{4\pi^2 \times 50^2C^2}\right)}} = 0.25$$

$$\therefore C = 4.0 \times 10^{-6}\ \text{F} = 4.0\ \mu F.$$

(*ii*) When an inductor of value L henry is put in series with the lamp,

$$\text{the impedance of the circuit} = \sqrt{(R^2 + \omega^2L^2)}$$

$$I = \frac{E}{\sqrt{(R^2 + \omega^2L^2)}}$$

$$\frac{200}{\sqrt{(80^2 + 4\pi^2 \times 50^2L^2)}} = 0.25$$

$\therefore$
$$L = 2.53\ \text{H}$$

(*iii*) When a resistance of $r\,\Omega$ is put in series with the lamp,

$$I = \frac{E}{R + r}$$

$$\frac{200}{80 + r} = 0.25$$

$\therefore$
$$r = 720\ \Omega$$

(*iv*) The insertion of a resistance of 720 Ω to run the lamp at its rated value will cause a dissipation of power. However, the use of pure inductance or of pure capacitance consumes no power. Therefore, it will be more economical to use inductance or capacitance.

13.7. The Transformer

It is a device for converting a low alternating voltage at high current into a high alternating voltage at low current and vice-versa. It is an electrical device based on the principle of mutual induction between the coils.

Construction. A transformer consists of two coils, called the *primary P* and *secondary S*, which are insulated from each other and wound on a common soft-iron laminated core (Fig. 13.15).

The alternating voltage to be transformed is connected to the primary while the load is connected to the secondary. Transformers which convert low voltages into higher voltages are called *step-up* transformers. Transformers which convert high voltages into lower voltages are called *step-down* transformers.

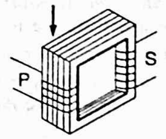


Fig. 13.15

∴ In a step-up transformer, the primary coil consists of a few turns of thick insulated copper wire of large current carrying capacity and secondary consists of a very large number of turns of thin copper wire. In a step-down transformer, the primary consists of a large number of turns of thin copper wire and the secondary of a few turns of thick copper wire.

Now when an ac is applied to the primary coil, it sets up an alternating magnetic flux in the core which also gets linked with the secondary. This change in flux linked with the secondary coil induces an alternating emf in the secondary coil. Thus the energy supplied to the primary is transferred to the secondary through the changing magnetic flux in the core.

Theory

(i) **Transformer on no load**

Let N_P and N_S be the number of turns in the primary and secondary respectively of the transformer (Fig. 13.16). When an alternating emf is applied across the primary, a current flows in its winding. This develops a magnetic flux in the core. Here it is assumed that there are no losses and no leakage of flux. Let ϕ = flux linked with each turn of either coil. This magnetic flux is linked up with both the primary and the secondary.

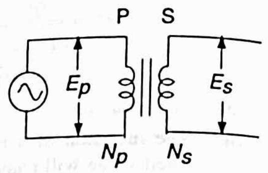


Fig. 13.16

By Faraday's law of electromagnetic induction, the emf induced in the primary is given by

$$\varepsilon_p = -\frac{d(N_p\phi)}{dt} = -N_p\frac{d\phi}{dt}$$

and the emf induced in the secondary,

$$\varepsilon_s = -\frac{d(N_s\phi)}{dt} = -N_s\frac{d\phi}{dt}$$

$$\therefore \quad \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}.$$

In an *ideal* transformer, the resistance of the primary circuit is negligible and there are no energy losses. So the induced emf ε_p in the primary is numerically equal to the applied voltage E_p across the primary. If the secondary is on open circuit, its resistance is infinite. So the voltage E_s across the terminals of the secondary is equal to the induced emf ε_s.

$$\frac{E_s}{E_p} = \frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} = K, \qquad \text{... (1)}$$

where K is called the *turns ratio* or *transformation ratio* of the transformer.

$$K = \frac{\text{voltage obtained across secondary}}{\text{voltage applied across primary}} = \frac{\text{No. of turns in secondary}}{\text{No. of turns in primary}}$$

In a step-up transformer $\varepsilon_s > \varepsilon_p$ and hence $N_s > N_p$.

In a step-down transformer $\varepsilon_s < \varepsilon_p$ and hence $N_s < N_p$.

Let I_p and I_s be the currents in primary and secondary at any instant.

In this case power output is equal to power input.

power in the secondary = power in the primary

$$E_s \times I_s = E_p \times I_p$$

$$\frac{I_p}{I_s} = \frac{E_s}{E_p} = \frac{N_s}{N_p} = K. \qquad \text{... (2)}$$

Thus, *when the voltage is stepped-up, the current is correspondingly reduced in the same ratio, and vice-versa.*

(ii) **Transformer on load**

If the primary circuit has an appreciable resistance R_p, the difference between the applied voltage E_p and the back e.m.f. ε_p must be equal to the potential drop $I_p \times R_p$ in the primary coil, *i.e.*,

$$E_p - \varepsilon_p = I_p \times R_p.$$

or
$$\varepsilon_p = E_p - I_p \times R_p \qquad \text{... (3)}$$

Again, if the secondary circuit is closed having finite resistance (load) R_s, a part of the induced emf ε_s in the secondary overcomes the potential drop $i_s \times R_s$ (Fig. 13.17). Hence the available P.D. across the secondary is given by

$$E_s = \varepsilon_s - I_s \times R_s$$

or
$$\varepsilon_s = E_s + I_s \times R_s.$$

Hence
$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{E_s + I_sR_s}{E_p - I_pR_p} = K$$

or $E_s + I_sR_s = K(E_p - I_pR_p)$

$$\therefore \quad E_s = KE_p - I_sR_s - KI_pR_p$$

$$= KE_p - I_sR_s - K^2I_sR_p \qquad (\because I_p = KI_s)$$

$$= KE_p - I_s(R_s + K^2R_p)$$

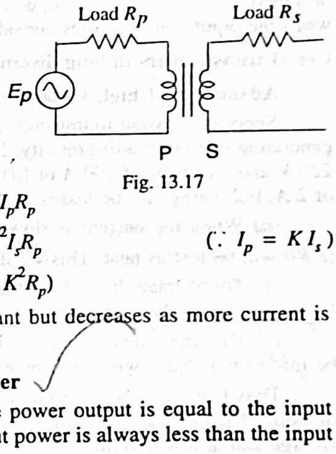


Fig. 13.17

In this case E_s/E_p is not a constant but decreases as more current is drawn from the secondary circuit.

Energy Losses in a Transformer

Only in an ideal transformer the power output is equal to the input power. In actual transformers the output power is always less than the input power because of unavoidable energy losses. These losses are :

(1) *Copper loss.* There is loss of power due to Joule heating in the primary and secondary windings.

(2) *Iron loss.* This is due to the eddy currents being produced in the core of the transformer. This is minimised by using a laminated iron core.

(3) *Hysteresis loss.* During each cycle of A.C., the core is taken through a complete cycle of magnetisation. The energy expended in this process is finally converted into heat and is, therefore, wasted. This is minimised by using silicon-iron for preparing the core. The hysteresis loop for this material is very narrow.

(4) *Leakage of magnetic flux.* Due to leakage, all the magnetic flux produced in the core by the primary is not linked with the secondary. They may pass through air. The loss due to this cause is minimised by using a shell type of core.

Uses of Transformers

(1) The *step-up* and *step-down transformers* are used in a.c. electrical power distribution for the domestic and industrial purposes.

(2) The *audio-frequency transformers* are used in radio receivers, radio-telephony, radio-telegraphy and in televisions.

(3) The *radio frequency transformers* are used in radio-communications at frequencies of the order of mega-cycles.

(4) The *impedance transformers* are used for matching the impedance between two circuits in radio communication.

(5) The constant current and constant voltage transformers are designed to give constant output current and voltage respectively even when the input voltage varies considerably.

Use of transformers in long distance power transmission :

Advantage of high voltage in transmission :

Suppose we want to transmit a given power (VI) = 44,000 W from a generating station to a distant city. It can be transmitted (i) at a voltage of 220 V and a current of 200 A or (ii) at a voltage of 22000 V and a current of 2 A. Following are the losses which occur in the transmission :

(a) When the current is flowing through the line wires, the energy (I^2Rt) will be lost as heat. This would be greater in the first case.

(b) The voltage drop along the line wire is equal to (RI). Again this loss is greater in the first case.

(c) The line wires, which are to carry the *high* current, will have to be made thick. Such wires will be expensive.

Thus from the above example it is clear that from the point of view of both efficiency and economy, the power must be transmitted at high voltage and at low current.

If we make use of D.C., transformation of voltage is not possible. If A.C. is used, voltage can be stepped up or stepped down by using transformers. Hence A.C. transmission is preferred to D.C. transmission.

The A.C. voltage generated at the power stations is stepped up by means of a transformer to 66,000 V and is transmitted to various sub-stations at the consumers' end. The sub-stations employ suitable transformers to step down the voltage to 440 for supplying to the industry and to 220 volt for domestic use (Fig. 13.18).

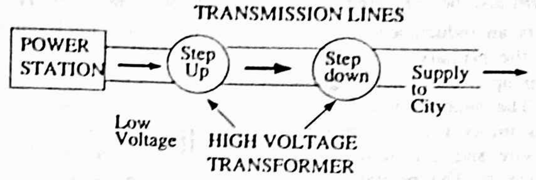


Fig. 13.18

13.8. Skin Effect

The current density j remains constant over any given section when a steady current passes through a uniform wire. But, when an alternating current of high frequency flows through a wire, the current density is not uniform at all points across a section. There is a greater current density at the surface layers than at the interior layers of the conductor. When the frequency is very large, the current is almost entirely confined to the surface layer. This phenomenon is called *Skin Effect.* Since the alternating currents of high frequency do not pass through the entire cross-section of the wire, the effective resistance of a wire for A.C. is much greater than that for D.C. Hence the conductor required to carry high-frequency alternating currents consists of a number of strands of fine wire connected in parallel at their ends, and insulated throughout their length from each other. This increases the surface area and thus decreases the resistance. Such a conductor has negligible Skin effect and its A.C. resistance is very nearly equal to its D.C. resistance.

Explanation. When the current passes along the axis of a cylindrical wire, the magnetic flux is finite. When the same current passes through the surface of the wire, the magnetic flux within the wire is zero. The current is changing with time with a fixed frequency over its entire cross-section. Hence the rate of change of flux in the wire (and the emf induced) is greater due to a current near the axis than that for a current near the periphery. As the induced e.m.f. opposes the applied e.m.f. the effective resistance or impedance is higher at the centre than in the outer layers. Hence less current passes through the inner layers than through the outer ones. Since the reactance is dependent on frequency ($X_L = \omega L = 2\pi\nu L$), the magnitude of the effect is greater at higher frequencies.